Atomic and nuclear physics

Course outline

- **Atomic and nuclear physics:**
- **Electronic devices**

Atomic and nuclear physics

Charged particles

Discharge tube

Cathode rays – production and properties, Positive rays – production and properties, Motion of cathode rays and beams in electric and magnetic fields, Specific charge – Thomson's experiment

Mass spectrometer (Bainbridge mass spectrometer), Millikan's oil drop experiment, The mole, Avogadro's number and faradays constant

Quantum theory

Photo electric effect, Characteristics of photo electric emission, Quantization of electromagnetic waves, Work function and threshold frequency, Einstein's equation, Experiment to verify Einstein's equation, measure planks constant, stopping potential., graph for variation of stopping potential and frequency, Applications, Rutherford's scattering experiment, Borh's atom, Stable electron energy levels, Emission and absorption spectra, Wave particle treatment of an electron, Ground, excited and ionization states, $\Delta E = hf,$

X- ray tube, x – rays, props, and radiation, characteristics, continuous (back ground), line x-ray spectra and cut off wave length, uses, diffraction, Bragg's law, hazards, safety precautions, numerical problems.

Nuclear physics

Atomic nucleus, constituents of the nucleus, atomic number and mass number, scientific representation of nuclide, isotopes and examples, unified atomic mass, equilvalence, Eisteins mass energy relation, binding energy and mass defect, binding energy per nucleon and its variation with mass number, significance, nuclear fission and fusion, numerical problems.

Radioactivity

Radiations emitted during radioactivity, decay equations, properties of radiations, detection, uses, laws, decay constant, $N = N_0e^{-\lambda t}$, decay curve, half- life, artificial radioactivity, applications of radioactivity, hazards and safety precautions.

Electronic devises

The c.r.o, applications, comparison with moving coil instruments, numerical problems.

Transistors, pn junction, including intrinsic and extrinsic conductors, doping, junction diode (rectification included), transistor characteristics, two transistor amplifier, logic gates and their combination Boolean algebra, numbers in base 2, 8, 10 and 12, solution to logic gates, numerical problems.

Cathode rays (electrons)

Cathode rays are streams or beams of fast moving electrons that are negatively charged. The passage of electricity through a gas takes place when the pressure of the gas is less than about 50mmHg, and in a neon lamp it is observed to be about 10mmHg.

Methods of production

(i) Using a discharge tube
(ii) Thermionic emission
(iii) Photo electric effect
(iv) Secondary emission
(v) Field emission

The gaseous discharge at various pressures

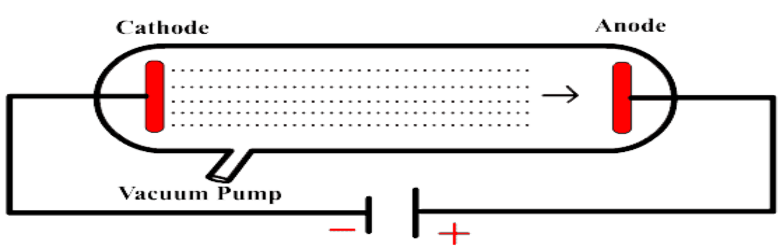
Discharge tube

The discharge tube is a sealed glass tube that contains air at a greatly reduced pressure (rarefied) to that of normal air pressure at sea level. The tube has a positive electrode (anode) at one end and a negative electrode (cathode) at the other.

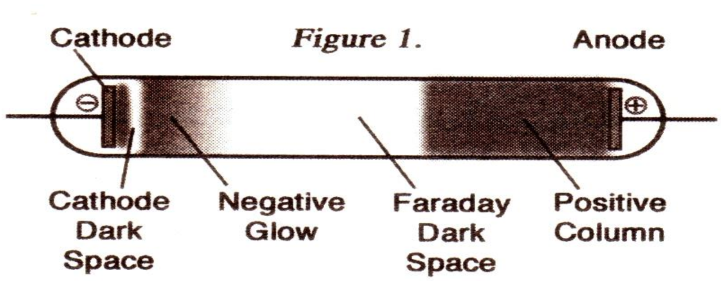
Action

When a large DC voltage is set up between the anode and cathode, fluorescence occurs within the discharge tube and the pattern of light seen depends on the air pressure within the tube.

When the air is pumped out to about 50mmHg/100mmHg, thin streamers of luminous gas appear between the anode and cathode (electrodes) as shown below.



At about 10mmHg, the discharge becomes a steady glow, spread through the tube as shown below.



The glow is broken up by two dark regions one nearest to the cathode called the cathode/Crookes dark space which is narrow and dark to see. Beyond Crookes, there is a bright region called the negative glow, and beyond that there is faraday dark space. Beyond faraday, there is a positive column which fills the rest of the discharge tube.

The negative glow and faraday dark space are clear. The cathode dark space is electrically most important, because the electrons from the cathode are accelerated by the electric field, gaining kinetic energy used to ionize gas atoms.

In the positive column, some atoms are being ionized by collision with electrons and others excited causing them to emit their characteristic spectra.

When the pressure of the gas in the discharge tube is reduced further, the dark spaces swell and positive column shrinks. At about 1mmHg, the cathode dark space becomes distinct, and at 0.1mmHg, it is several centimeters long.

Eventually, as the pressure falls to about 0.01mmHg, the cathode dark space stretches from the anode to the cathode and the negative glow and positive column varnish (disappear).

When the whole cathode dark space occupies the whole discharge tube, the walls of the glass tube fluorescence. The electrons moving around the space and hitting glass are called cathode rays, when they strike the anode, they produce x-rays.

N.B:

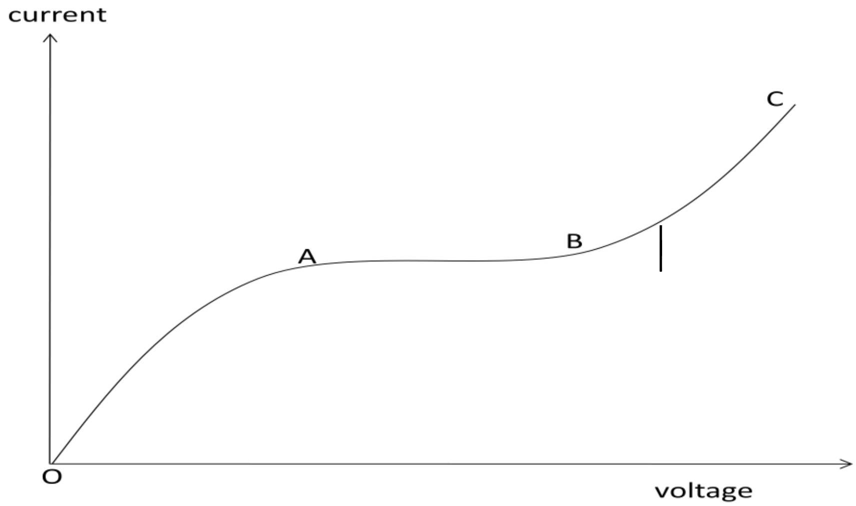
The discharge tube is not a convenient one for producing and studying cathode rays/electrons because:

i. A gas is needed at an appropriate pressure.
ii. A very high p.d is needed across the tube.
iii. X-rays are produced which may be dangerous.

Applications of discharge tube

i. Neon signs
ii. Making mercury lamps and sodium lamps
iii. Street lights / flood lights
iv. Fluorescent tubes

Graph of current against voltage for a discharge tube



Explanation

OA- very weak current detected due to release of electrons by the cathode because of external radiation.

AB- as the voltage increases, the anode attracts more electrons from the cathode thus the current increases.

BC – the accelerated electrons from the cathode collide with the gas atoms thus releasing more electrons. There is rapid and uncontrolled increase in the current.

Question

1. Give an account of the stages observed when an electric discharge passes through a gas at pressures varying when 50mmHg atmospheric pressure to about 0.01mmHg as air is pumped out when the potential difference across the tube maintained at extra high tension.
2. Explain why the discharge tube is not a convenient way of producing cathode rays.
3. Give an account of the stages observed when an electric discharge passes through a gas at pressures varying from atmospheric to about 0.01mmHg as air is pumped out when the p.d across the tube is maintained at extra high tension.
4. State two disadvantages of discharge tubes when used to study cathode rays.

Effect of Low Pressure in the Discharge Tube

When the gas pressure is very low there are few gas atoms in the discharge tube and therefore there is no hindrance to the movement of electrons thus the gas conducts electricity.

Production of cathode rays using a cathode ray tube

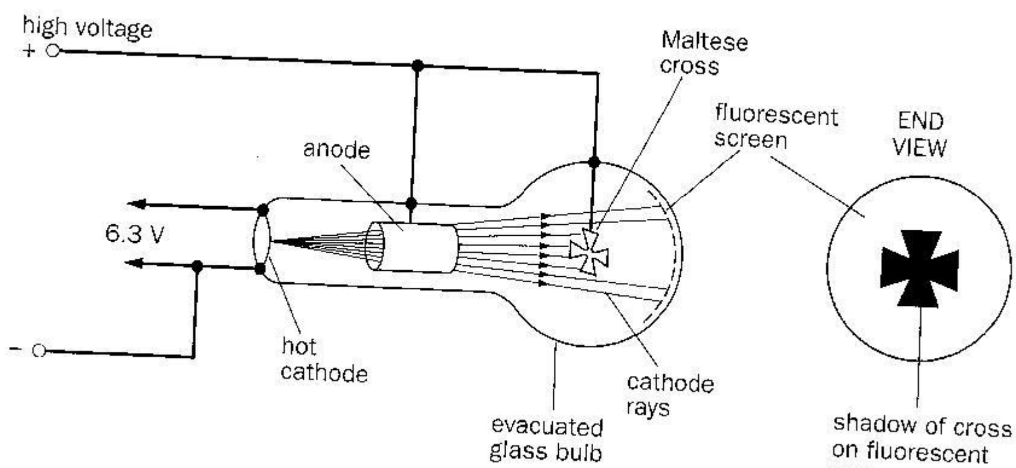
Structure

Production

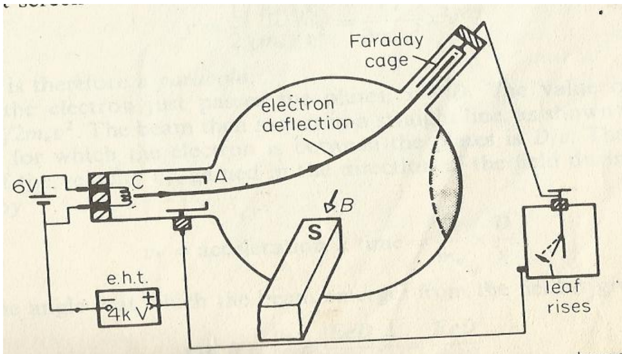
The cathode is heated by a low p.d and produces electrons by thermionic emission. The electrons are focused by cathode and accelerated by extra high tension to fluorescent screen which gives a glow when they strike the screen. The fast moving electrons constitute cathode rays.

Properties of cathode rays

1. They travel in straight lines.
The cathode is heated by a low p.d and emits electrons by thermionic emission. They are accelerated by the high p.d connected across the anode. A maltese cross is placed in their paths and a sharp shadow of a maltese cross is casted onto the screen. This shows that they travel in straight line.



2. Cathode rays carry a negative charge. This is confirmed by collecting electrons inside a Perrin tube, shown below.



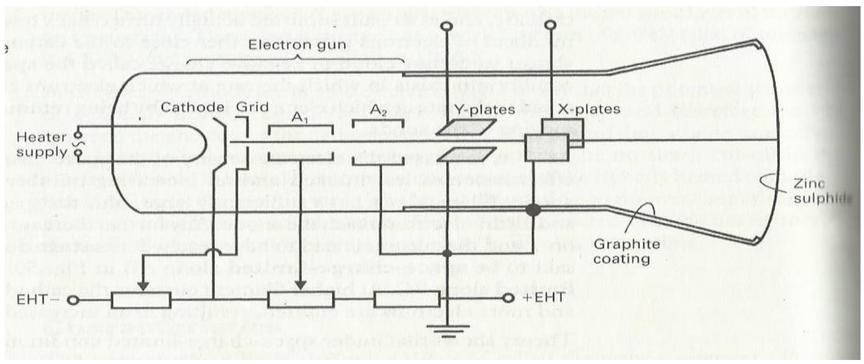
The electrons are deflected by the magnet S until they pass into the **faraday cage.** The cage is connected to the plate of an electroscope, which has been negatively charged using an ebonite rod and fur. As soon as the electrons are deflected into the cage, the leaf rises further, showing that an extra negative charge has been collected by the cage. This supports the idea that cathode rays are electrons.

3. Carry energy by the virtue of its mass and velocity. Cathode rays set a paddle wheel into motion when it is placed in the path of these rays one the bladder of the paddle wheel.
4. Cathode rays heat the object only which they fall. The cathode ray particles possess kinetic energy. When these particles strike an object, a part of the kinetic energy is transferred to the object. The causes a rise in the temperature of the object.
5. Cathode rays cause green fluorescence on glass surface, i.e., the glass surface only which the cathode rays strike show a colored shine.
6. Deflected by both electric and magnetic fields.
7. Cathode rays can penetrate through thin metallic sheets.
8. Cause x rays when stopped by matter. Cathode rays when fall only certain metals such as copper, but rays produced. The X-rays are not deflected by electrical or magnetic fields. X-rays pass through opaque materials such as black paper, but stopped by solid objects such as bones.
9. Cathode rays travel with speed nearly equal to that of light.
10. Cathode rays ionize the gases through which they travel.

Questions:

1. What are cathode rays.
2. Describe an experiment to show that cathode rays travel in straight lines.
3. State and justify two properties of cathode rays.

The cathode ray oscilloscope (C.R.O)



Structure

It consists of an electron gun, deflecting system and fluorescence screen. The electron gun consists of the cathode, grid and anodes A_1 and A_2. The cathode emits electrons thermionically. The grid controls the number of electrons while anodes A_1 and A_2 accelerate the electrons to high kinetic energy so that they cause light emission when they hit the screen.

The deflecting system consists of horizontal y – plates and vertical x- plates. The x- plates deflect the beam horizontally while the y- plates deflect the beam vertically. The screen is coated with zinc sulphide which absorbs the kinetic energy of the bombarding electrons and emits electrons.

Action

The low voltage supplies current that heats the filament which in turn heats the cathode. Thermionic emission takes place at the cathode and the emitted electrons are accelerated and focused by the anodes which have increasing potential relative to the filament. These electrons are deflected horizontally by x- plates and vertically by y- plates and strike the screen where they form a green spot. The brightness of the spot is controlled by the control grid which allows more or few electrons to reach.

N.B: When the electrons hit the zinc sulphide, they knock electrons from it. These are called secondary electrons and they are collected by graphite coating where they flow to the earth.

Measurement of voltage of the battery

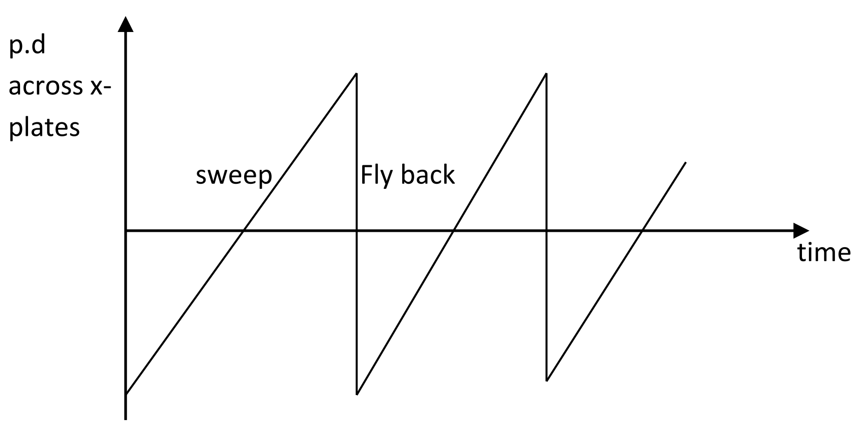
The low voltage supplies current that heats the filament which in turn heats the cathode. Thermionic emission takes place at the cathode and the emitted electrons are accelerated and focused by the anodes. If no p.d across the plates, the electrons form a spot on the screen whose position is noted. The battery is connected across the y- plates and the y-sensitivity a noted. The new position of the spot is noted and the deflection d, calculated. The p.d of the battery is given by ad.

TIME BASE

Time base: Is a saw tooth oscillator.

When the time base is on, an internally generated saw tooth voltage is applied to the x-plates. This sweeps the electron beam from left to right at a constant rate. For a particular sweep rate is determined by the setting of the time base control.

The saw tooth then returns the beam to the initial position at the extreme left of the screen almost instantaneously. The time base for this right to left sweep is called the fly back time. The brightness is automatically reduced to zero at the start of the fly back.



Use of the time base in the cathode ray oscilloscope.

It provides a saw tooth voltage that sweeps the electron spot from the left to the right of the screen at a steady speed. Consequently, it helps in studying the variation with time.

Uses of the C.R.O

i. Used to measure direct and alternating current.
ii. Used to measure frequency.
iii. Used to measure phase differences.
iv. Used to measure small time intervals.

Advantages of the C.R.O over a voltmeter

It has nearly infinite resistance to d.c and very high impedance to a.c.
It has instantaneous response.
It has no coil to burn out.
It can be used to measure both a.c and d.c.

Measurement of frequency using a C.R.O

The signal whose frequency is required is applied to the y- plates and the trace is obtained. The unknown frequency can be obtained by comparing it with a known one. If in one cycle of a signal of unknown frequency f_1 occupies a horizontal distance d_1, then $f_1 \alpha d_1 \Rightarrow f_1 = kd_1$.....1 where k is a constant known as frequency sensitivity.

With the same base setting, one cycle of the signal of known frequency f_2 occupies a distance d_2, then $f_2 = kd_2$..........2 . 1 divide by 2

$\frac{f_1}{f_2} = \frac{d_1}{d_2} = \frac{T_2}{T_1}$, where T is the period

Measurement of A.C (sinusoidal) voltage

An unknown A.C voltage whose peak value is required is connected to the y- plates. With the time base switched off, the vertical line on the screen is centered and its length then measured. This is proportional to twice the amplitude or peak voltage V_0. ($l \; \alpha \; 2V_0$)

By measuring the length corresponding to a known A.C voltage v, then V_0 can be found. ($v \; \alpha \; l_2$)

Thus $l_1 = 2kV_0$ ….1, where k is a constant called voltage sensitivity. And $l_2 = kv$.....2

Dividing the equations we have,

$\frac{2V_0}{v} = \frac{l_1}{l_2} \Rightarrow V_0 = \frac{1}{2}\frac{l_1}{l_2}v$, thus the peak voltage can be obtained.

N:B

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

Example

1. A C.R.O has its sensitivity set to $10Vm^{-1}$. A sinusoidal in put voltage is suitably applied to give a steady trace with the time base switched on so that the electron beam takes 0.01s to traverse the screen. If the trace seen has a peak to peak height of 4cm and contains two complete cycles, calculate the:

(a) Root mean square value of the input voltage.

(b) Frequency of the input signal.

Solution

(a) $peak\ to\ peak\ voltage = 4 \times 10 = 40V,$

$$peak\ voltage = \frac{peak\ to\ peak}{2} = \frac{40}{2} = 20V$$

$$V_{r.m.s} = \frac{peak\ voltage}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.14V$$

(b) Using : $T = \frac{t}{2} = \frac{0.01}{2} = 0.005s$, and $f = \frac{1}{T} = \frac{1}{0.005} = 200Hz$

Measurement of phase

With the time base switched off, one in put is connected to the x plates and the other to the y plates. We consider only the case when the frequencies of the two signals are the same. An ellipse will be seen on the screen. The trace is centered and the peak vertical displacement y_2 in the middle and the peak vertical displacement y_1 of the ellipse are both measured in the y- direction. Suppose the x- displacement is given by $x = a\sin\omega t$, where a is the amplitude in the x direction and the y displacement $y = y_1 \sin(\omega t + \vartheta)$, where y_1 is amplitude in y direction and ϑ is the phase angle.

When x = 0, sinωt = 0, so ωt = 0. In this case, $y = y_2 = y_1 \sin\vartheta$, hence $\sin\vartheta = \frac{y_2}{y_1}$ from which ϑ can be measured.

Measurement of small time intervals

If two events which are separated by a small time interval are each caused to produce a voltage pulse on a C.R.O, the horizontal separation of the pulse can be used together with the value of the time base setting to estimate the time interval.

Example

1. A resistor of 1000Ω carries an alternating current. When the oscilloscope is used to measure the p.d across it, the reading is 20.0Vr.m.s. When an a.c voltmeter is used, the reading is 18.0V. What is the resistance of the voltmeter?

Solution:

Oscilloscope: this gives the accurate value since it draws no current.

$$I = \frac{V}{R} = \frac{20}{1000} = 0.02A.$$

Voltmeter: this gives inaccurate value of p.d because some current is diverted through its resistance R which is in parallel with 1000Ω resistor.

Current in 1000Ω resistor $= \frac{18}{1000} = 0.018$, and through the voltmeter, current $I = \frac{18}{R}$, where R is the resistance of the voltmeter, hence total current

$$\frac{18}{R} + 0.018 = 0.02 \Rightarrow \frac{18}{R} = 0.002 \Rightarrow R = 9000\Omega$$

Exercise

1. An a.c voltmeter and C.R.O are used in turn to measure the p.d across a resistance of 10000Ω in which there an a.c. The readings are 9.0V and 10.0Vr.m.s respectively. Calculate the resistance of the voltmeter. (Ans: 90000Ω). What is the advantage of using a C.R.O for this type of measurement?

2.(a) A C.R.O has its y – sensitivity set to $25Vcm^{-1}$. A sinusoidal input voltage is suitably applied to give a steady trace with time base switched on so that the electron beam takes 0.001s to traverse the screen. If the trace seen has a peak to peak height of 6.0cm and contains four complete cycles, find the

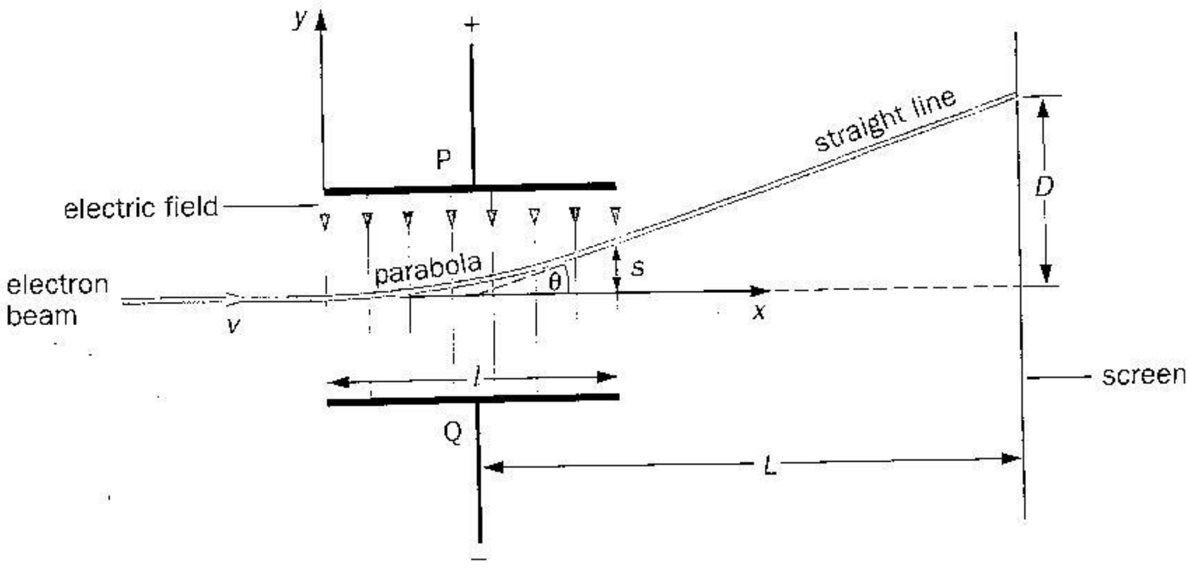
(i) R.m.s value of the input voltage

(ii) Frequency of the input signal

(b) Describe with the aid of diagram, the structure and made of operation of cathode ray oscilloscope (C.R.O)

Electron motion in electric and magnetic field

Motion in an electric field



Suppose a horizontal beam of electrons, moving with velocity *u*, passes between two parallel plates. If the p.d between the plates *v* and their distance apart is *d*, the field intensity $E = \frac{v}{d}$. Hence the force on an electron of charge *e* moving between the plates $= Ee = \frac{ev}{d}$ and is directed towards the positive plate.

By Newton's second law of motion, $F = ma = eE = \frac{ev}{d} \Rightarrow ma = \frac{ev}{d} \Rightarrow a = \frac{ev}{md} = \frac{eE}{m}$

Since the electricity intensity *E* is vertical, no horizontal force acts on the electron entering the plates.thus the horizontal velocity *V*, of the beam is un affected.

In vertical direction the displacement, $y = \frac{1}{2}at^2$, where t is the time and a is the acceleration

There fore $y = \frac{1}{2}\frac{Ee}{m}t^2$..............................1

In the horizontal direction, the displacement $x = ut \Rightarrow t = \frac{x}{u}$, substitung t in 1 gives

$y = \frac{1}{2}\left(\frac{Ee}{m}\right)\frac{x^2}{u^2} = \frac{Ee}{2mu^2}x^2$. The path is there fore a parabola.(*x = D)*

Thus the component of the velocity V_1, gained in the direction of the field during this time, is given by $V_1 = at = \frac{Eex}{mu}$, hence the angle θ at which the beam emerges from the field is given by $\tan\theta = \frac{V_1}{u} = \frac{Eex}{mu}.\frac{1}{u} = \frac{Eex}{mu^2}$

Velocity gained by the electron beam as it emerges out of the region between the plates, $v = \sqrt{v_1^{\ 2} + u^2}$

Question: Describe the motion of an electron inside the region between two oppositely charged plates.

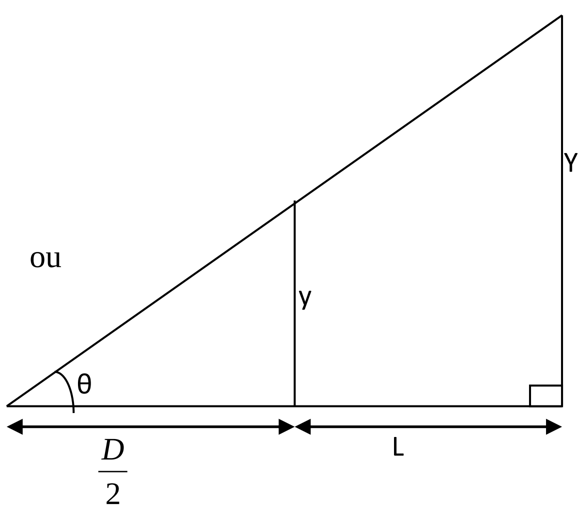
Solution:

The electrons experience an electric force in a direction opposite to that of the field. This causes them to describe a parabolic path.

Motion outside the field:

The electron beam continues in a straight path with constant force since the electric force on it is zero.

Also:



$$\tan\theta = y \div \frac{D}{2} = \frac{yD}{2} = \frac{Y}{\frac{D}{2}+L}$$, where L is the distance of the screen from the plates.

N.B:

kinetic energy of the electrons = $\frac{1}{2}mu^2 = ev_a, v_a =$ accelerating p.d, then $u = \sqrt{\frac{2ev_a}{m}}$

Question: describe the motion of an electron beam outside the region of plates.

Solution:

The electron beam continues in a straight path with constant force since the electric force on it is zero.

Example:

1. A beam of electrons moving with a velocity of $2.0x10^7ms^{-1}$ enters mid way between two horizontal plates P and Q in a direction parallel to the plates. P and Q are 10cm long and 4cm apart, and have a p.d, v between them. Calculate the value of v.
Solution

2. A beam of electrons is accelerated through a p.d of 500v and then enters a uniform electric field of strength $3.0x10^4vm^{-1}$, created by two parallel plates of length 4cm. calculate:
   i. Speed v of the electrons as they enter the field.91.33x10^7)
   ii. Time that each electron spends in the field.(3x10^{-9})
   iii. The angle through which the electrons have been deflected by the time they emerge out of the field.(50^0)
   iv. Speed of the electrons as they leave the region between the plates.
3. Two horizontal parallel plates are each 5cm long and 1.5cm apart with a potential difference of 240V maintained between them. Electrons are accelerated to an energy of 2.0KeV enters mid way between the plates from one edge. Calculate the:
   (i) Vertical displacement at the other edge of the plates.
   (ii) Vertical displacement on a screen placed 30cm from the edge of the plates.
4. An electron of charge *e,* and mass *m,* is emitted from a hot cathode and then accelerated by an electric field towards the anode. If the potential difference between the cathode and the anode is *V,* show that the speed of the electron, *u,* is given by $u = \sqrt{\frac{2ev}{m}}$
5. An electron of energy 8Kev enters mid way between two horizontal plates each of length 5.0cm and separated by a distance of 2.0cm. A p.d of 30V is applied across the plates. A flourescence screen is placed 15cm beyond the plates. Calculate the:
   i. Speed of the electrons as they enter the region between the plates.
   ii. Vertical deflection of the electron on the screen.
6. An electron starts from rest and moves in an electric field intensity of $2.4x10^3Vm^{-1}$. Find:

i. Force on the electron.
ii. Acceleration of the electron.
iii. Velocity acquired in moving through a p.d of 90V.

7. A beam of electrons each of mass, *m*, and charge, *e*, is directed horizontally with speed *u*, into an electric field between two horizontal metal plates separated by a distance, *d*.
   i. If the p.d between the plates is *V*, show that the deflection *y* of the beam is given by $y = \frac{1}{2m}\left(\frac{eV}{du^2}\right)x^2$, where *x*, is the horizontal distance travelled.
   ii. Explain the path of the electron beam as it emerges out of the electric field.

8. A beam of electrons is accelerated through a p.d of 2000V and is directed mid- way between two horizontal plates of length 5.0cm and separation 2.0cm. the p.d across the plates is 80V.
   i. Calculate the speed of the electrons as they enter the region between the plates.
   ii. Explain the motion of the electrons between the plates.
   iii. Find the speed of the electrons as they emerge from the region between the plates.

9. Electrodes are mounted at opposite ends of a low pressure discharged tube and a potential difference of 1.20KV applied between them. Assuming that the electrons are accelerated from rest. Calculate the maximum velocity which they would require (specific electron charge = 1.76×10^{11} Ckg^{-1}.

Also: gain in kinetic energy = $\frac{1}{2}mv_y{}^2, v_y = at = \frac{xeE}{u}$, hence

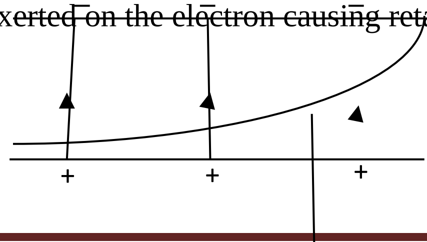
Gain in kinetic energy $= \frac{m}{2}\left(\frac{eEx}{u}\right)^2$, Also change in kinetic energy = work done by the electric field against charge or

Loss in kinetic energy = work done by the charge against the electric field, thus

$\frac{1}{2}mv_y{}^2 = QV$ or eV, for an electron

For a stationery electron in an electric field, $F_e = eE = mg$ or $QE = mg$

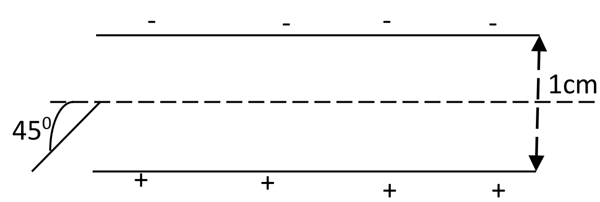
Note: for an electron projected towards a negatively charged plate, a repulsive force will be exerted on the electron causing retardation (change direction)



Retardation force = $-eE = ma \Rightarrow a = -\frac{eE}{m}$

Example:

1. Two large oppositely charged plates are fixed 1cm apart as shown below. The p.d between the plate is 50V.



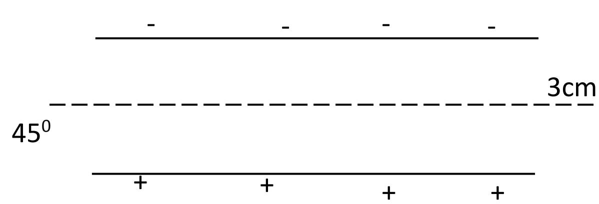
An electron beam enters a region between the plates at an angle of 45^0. Find the maximum speed the electron must have in order for them, not to strike the upper plate. (electron mass = $9.11x10^{-31}$).

Solution

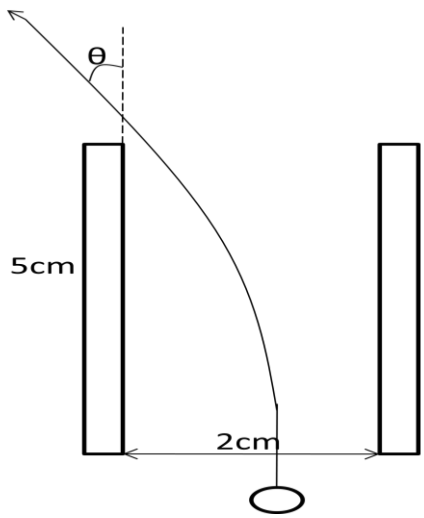
for the electron not to strike the upper plate, then the distance travelled should be half the separation of plates.

Question

1. The figure below shows an electron beam directed midway between oppositely parallel charged plates separated by a distance of 3cm. If the p.d of 46V is applied across the plates, find the maximum speed which the electrons must travel in order not to strike the upper plate. ($4.02x10^6$)



2.

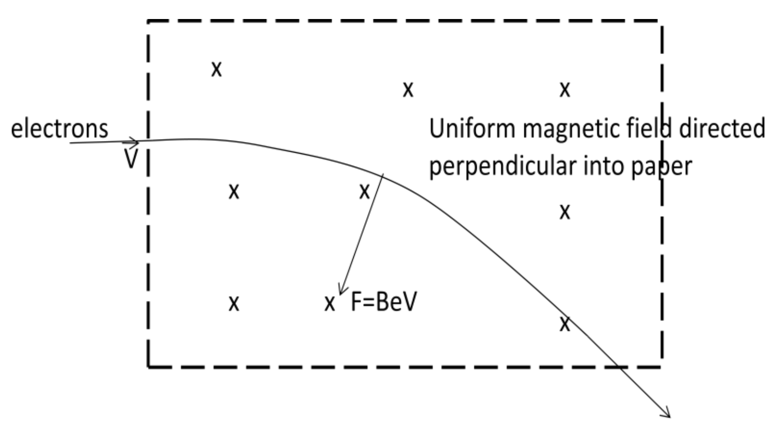


Two vertical parallel metal plates of length 5cm are separated by 2cm. A beam of doubly ionized particles travelling at a speed of $1 \times 10^5 ms^{-1}$ enter symmetrically between them. When a pd of 33.2V is applied between the plates, the beam just misses one of the plates as it emerges out at an angle θ. Calculate the;

(i) The mass of the particles in unified atomic mass unit (U). Ans: 4U

(ii) Value of θ. Ans: 21.8^0

Motion of an electron in a magnetic field



The electron experiences a constant magnetic force , F = BeV. The force is perpendicular to the direction of V and the field B, according to the Fleming's left hand grip rule. The resultant motion of the electrons is a circular path of radius, r, given by $BeV = \frac{mV^2}{r} \Longrightarrow r = \frac{mV}{Be}$. Thus motion is in circular path within the field.

NB:

The magnetic force cannot change the energy (speed or kinetic energy) but just deflects the electron. The force Bev is always normal to the path of the beam. If the field is uniform, the force is constant in magnitude and the beam then travels in a circle of radius r. Then, $Bev = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Be} = \frac{momentum}{Be}$

If the velocity v of the electron reduces continuously due to collisions, then, the radius of its path decreases and the electron will thus tend to spiral instead of moving in a circular path of constant radius.

Question:

1. *Describe the motion of an electron in a magnetic field of flux density B teslar.*
2. *Show that the path described by an electron beam moving with speed, V in uniform magnetic field of flux density, B acting perpendicular to the direction of motion is a circle.*
3. *A beam of positive ions moving with velocity v enters a region of uniform magnetic field of flux density B with a velocity at right angles to the field. By use of a diagram, describe the motion of the ions.(replace electrons with ions and e with Q)*

Example:

1. An electron is moving in a circular path at $3.0 \times 10^6 ms^{-1}$ in a uniform magnetic field of flux density 4×10^{-4}T. Find the radius of the path.

   Solution

$$BeV = \frac{mV^2}{r} \Longrightarrow r = \frac{mV}{Be} = \frac{3.0 \times 10^6}{0.0004 \times 1.76 \times 10^{11}} = 0.043m$$

2. An electron having 450eV of energy moves at right angles to a uniform magnetic field of flux density 1.50×10^{-3}T. Show that the path of the electron is a circle and find its radius. (Assume specific charge of the electron is $1.76 \times 10^{11} Ckg^{-1}$).
   Solution
   The kinetic energy of the electron is given by $\frac{1}{2}mv^2 = eV$, where V is 450V and m is mass of electron, thus $v = \sqrt{\frac{2eV}{m}}$

The path of the electron is a circle because the force Bev is constant and always normal to the electron path. Its radius is given by $Bev = \frac{mv^2}{r} \Rightarrow r = \frac{m}{e}.\frac{v}{B} = \frac{m}{e}.\frac{1}{B}\sqrt{\frac{2eV}{m}}$

$= \frac{1}{B}\sqrt{2\frac{mV}{e}}$ but $\frac{e}{m} = 1.76x10^{11}, V = 450$V, B = 1.5x$10^{-3}$T

Therefore, $r = \frac{1}{1.5x10^{-3}}\sqrt{\frac{2x450}{1.76x10^{11}}} = 4.8x10^{-2}\,m$ or 4.8cm

3. A beam of electrons having a common velocity enters a uniform magnetic field in a direction normal to the field.

   i. Describe and explain the subsequent motion of the electrons.

   Solution

   The electrons will describe a circular path since they experience a magnetic force at right angles to the direction of motion and to the field, this provides the centripetal force.

   ii. Explain whether a similar path would be followed if a uniform electric field was substituted for a magnetic field.

   Solution

   The electrons experience an electric force in a direction opposite to that of the field. This causes them to describe a parabolic path.

Exercise

1. Protons with a charge – mass ratio of 1.0x10^8Ckg^{-1}, are rotated in a circular orbit of radius r when they enter a uniform magnetic field of 0.5T. Show that the number of revolutions per second, f, is independent of r and calculate f.
2. A beam of positive ions is accelerated through a p.d of 1000V into a region of uniform magnetic field of flu density 0.4T. While in the magnetic field, it moves in a circle of radius 2.4cm. Derive an expression for charge to mass of the ions, and determine its value.
3. A beam of cathode rays is directed mid way between two parallel metal plates of length 4cm and separation 1cm. the beam is deflected through 10cm on the fluorescent screen placed 20cm beyond the edge of the plates when a p.d of 200V is applied or cancelled by a magnetic field of 1.4x10^{-3}T applied normal to the electric field between the plates. Find the ration of mass to charge of cathode rays.

7 An electron mass *m* and charge, *e* travels with a speed of *u* in a circular path of radius r of a plane perpendicular to a uniform magnetic field of flux density B.

(i) Write down an algebraic equations relating the centripetal and magnetic force acting on the electron and define all the symbols used.

(ii) Show that the periodic time of this electron is given by $T = \frac{2\pi m}{Be}$

(iii) If the speed of the electrons changed to 2v, what will be the effect, if any on the orbital radius.

Crossed fields

If a uniform electric field and a uniform magnetic field are perpendicular to each other in such a way that they produce deflections in opposite directions, they are called ***crossed fields***. If the forces exerted by each of the fields are the same in magnitude, then $F_e = F_m$

$$\Rightarrow eE = BeV \Rightarrow V = \frac{E}{B}$$

Example:

1. An electron accelerated by a p.d of 1000V passes through a uniform electric field of intensity, E, crossed with a uniform magnetic field of flux density 0.3T. If the electron emerges un deflected, calculate the electric field intensity.

2. Electrons are accelerated through a potential of $2.0\text{x}10^3$V and passes at right angles into a uniform magnetic field of flux density $1.0\text{x}10^2$T.

   i. Find the radius of the path.
   ii. Calculate the field intensity of an electric field to be crossed with the magnetic field above so that the electrons continue moving in a straight line.

3. A beam of electrons is fired into a uniform electric filed. The length of the plates is 10cm and plate separation is 5cm. when a constant p.d of 720V is applied across the deflecting plates, the beam is deflected vertically at the edge of the plate by a distance of 2.5cm. The beam is straightened out by applying a uniform field of strength $6.5\text{x}10^{-4}$T perpendicular to the beam with the same p.d. Calculate the:

   (i) Speed of the beam entering the fields.
   (ii) Specific charge of the electrons.

SPECIFIC CHARGE

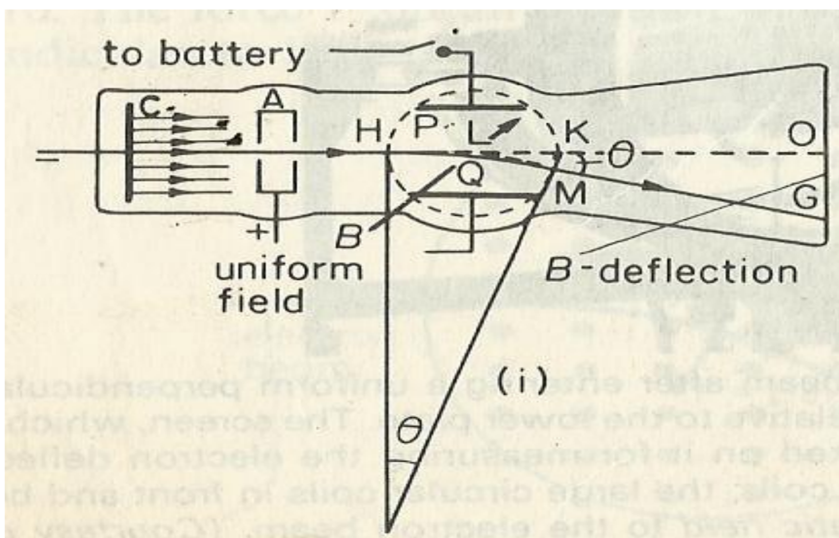
Specific charge is the ratio of charge of an electron to its mass. S.I unit is coulomb per kilogram (Ckg^{-1})

Determination of specific charge ($\frac{e}{m}$)

There are two methods used which include J.J Thomson experiment (use of crossed fields) and fine beam tube.

a. J.J (Joseph John) Thomson experiment

It consists of a cathode C and anode A with slits cut in opposite directions so that the



Electrons passing through are limited to narrow beam.

Action

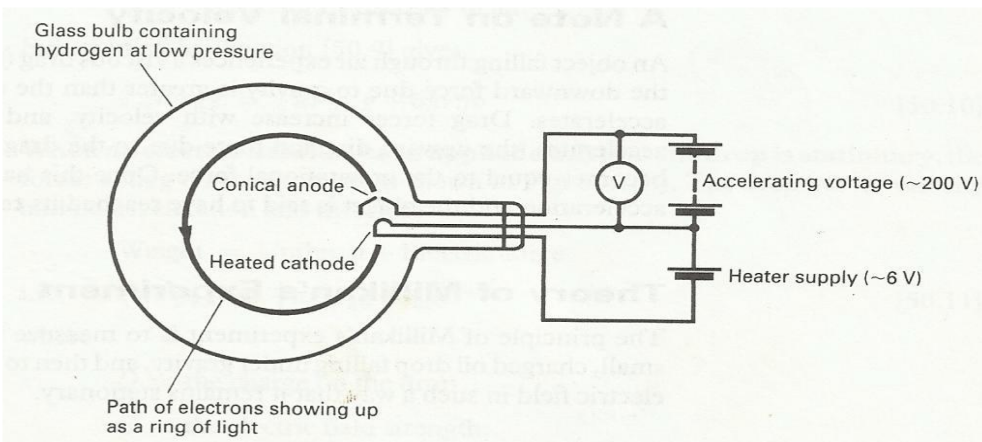
Electrons are emitted from the filament thermionically and accelerated to anode A. With no emf applied, the electrons will then strike the screen at O, which is noted.

A magnetic field of known flux density B is applied at X to deflect the electron beam to point G, thus $BeV = \frac{mV^2}{r} \Rightarrow \frac{e}{m} = \frac{V}{rB}$.

An electric field is also applied simultaneously at x and adjusted until the beam goes back to O, hence $BeV = eE \Rightarrow V = \frac{E}{B}$, which is the velocity of the electron beam.

The radius r of the path described is determined by using the relation, $\tan\theta = \frac{HK}{r} = \frac{OG}{KO} \Rightarrow r = \left(\frac{HK}{OG}\right)KO$

b. **Using a fine beam tube**



The fine tube is a glass containing hydrogen at low pressure.

Action

The electrons are emitted thermionically from the cathode and are accelerated to the anode.

These electrons collide with hydrogen atoms causing the atoms to emit light, thereby revealing the circular path of electrons which shows up as a luminous ring.

A uniform magnetic field, B within the bulb deflects the beam back to the cathode.

The radius of the path is measured by use of a plane mirror.

The specific charge is calculated from $\frac{e}{m} = \frac{2V}{B^2 r^2}$.

Example

1. A narrow beam of electrons is emitted from a small electron gun within a glass bulb that contains enough gas to glow where the beam strikes. Using 300V to accelerate the electrons, a uniform magnetic field of $7.3x10^{-4}$T produces a circle of glow with a radius of 8.0cm. Obtain a value for specific charge of the electrons.
   Solution

2. A beam of cathode rays is directed mid way between two parallel metal plates of length 4cm and separation 1cm. the beam is deflected through 10cm on a fluorescent screen placed 20cm beyond the nearest edge of the plates when a p.d of 200V is applied across the plates. If this deflection is cancelled by the magnetic field of B = $1.14x10^{-3}$T applied normal to the electric field between the plates. Find the ratio of charge to mass of the cathode rays.
   Solution

3. In the C.R.O, an electron beam passes between the y- deflector plates each 5cm long and 0.5cm apart. The distance between the centre of the y- plates and the screen is 20cm and the p.d between the anode and the electron gun is 250V. Determine the deflection in volts per metre of the electron beam on the screen of the C.R.O.

4. (a) Describe an experiment to determine the magnitude of the charge associated with an electron.

   (b) State the nature of the path traversed by a charged particle when it is projected at right angles to (i) a uniform magnetic field, (ii) a uniform electric field.

   c) A uniform magnetic field and a uniform electric field are superimposed so that they allow a charged particle of velocity v to proceed in a straight line in a vacuum.

   d) Explain the relations between (i) the directions of the fields and of the particle velocity, (ii) the magnitudes of the fields.

   e) Two plane metal plates 40 cm long are held horizontally 3.0 cm apart in a vacuum, one being vertically above the other. The upper plate is at a potential of 300 volts and the lower is earthed Electrons having a velocity of 1.0×10^7 m s^{-1} are injected horizontally midway between the plates and in a direction parallel to the 40cm edge. Calculate the vertical deflection of the electron beam as it emerges from the plates, (e/m for electron = 1.8 x 10" C kg^{-1}

5. a) Show that if a free electron moves at right angles to a magnetic field the path is a circle.

   b) Show also that the electron suffers no force if it moves parallel to the field Point out how the steps in your proof are related to fundamental definitions.

   c) If the path of the electron is a circle, prove that the time for a complete revolution is independent of the speed of the electron.

d) In the ionosphere electrons execute 1.4 x 10^6 revolutions in a second. Find the strength of the magnetic induction B in this region.

6. When low energy electrons are moving at right angles to a uniform magnetic field of flux density 10^{-3}Wb m^{-2}, they describe circular orbits 2.82 x 10^7 times per second. Deduce a value for specific charge.

7. A uniform magnetic field and a uniform electric field are superimposed so that they allow a charged particle of velocity v to proceed in a straight line in a vacuum.

Explain the relations between (i) the directions of the fields and of the particle velocity, (ii) the magnitudes of the fields.

8. Two plane metal plates 40 cm long are held horizontally 3.0 cm apart in a vacuum, one being vertically above the other. The upper plate is at a potential of 300 volts and the lower is earthed Electrons having a velocity of 1.0 x 10^7 m $s^{-1}$are injected horizontally midway between the plates and in a direction parallel to the 40 cm edge. Calculate the vertical deflection of the electron beam as it emerges from the plates, (e/m for electron = 1.8 x 10" C kg^{-1}.)

9. In the ionosphere electrons execute 1.4 x 10^6 revolutions in a second. Find the strength of the magnetic induction B in this region. (Mass of an electron = 9.1 x10^{-28}g; electronic charge = l.6x 10^{-19} coulomb.)

10. Give a short account of the phenomena observed when an electric discharge passes through a gas at very low pressure (10.6atmosphere). Describe very briefly experiments which reveal the nature of the discharge.

11. What is the direction of the force acting on a negatively charged particle moving through a magnetic field? Deduce the shape of the path of a charged particle projected at right angles to a uniform magnetic field.

12. An ion, for which the charge per unit mass is 4.40 x 10^7 C kg^{-1}has a velocity of 3.52 x 10^7 cm $s^{-1}$and moves in a circular orbit in a magnetic field of induction0.4 Wb m^{-2}. What will be the radius of this orbit?

14. Give an account of the structure of atoms including the significance of the terms atomic mass and atomic number. State how the fundamental particles of which atoms are composed differ from each other as regards mass and electric charge.

15. A potential difference of 600 V is maintained between two identical horizontal metal plates placed 4.0 cm apart one above the other in an evacuated vessel. Particles each with mass 9.1 x 10^{-31} kg and electric charge 1.6 x 10^{-19} C are emitted with negligible velocity from the plate at the lower potential. For one of the particles calculate

(a) the ratio of the electric force to the gravitational force on it.

(b) its acceleration.

(c) the kinetic energy it acquires on reaching the other plate. (Assume g = 10 m s^{-2}.)

Millikan's oil drop experiment

Theory of Millikan's experiment

Millikan first measured the terminal velocity of an oil drop through air. He then charged the oil drop and applied an electric field to oppose gravity. The drop then moved with a different terminal velocity, which was again measured. Suppose the radius of the oil drop is *a*, the densities of oil and air are ρ and δ respectively, and the viscosity of air is η. When the drop, without charge, falls steadily under gravity with terminal velocity v_1, the upward viscous force $= 6\pi\eta a v_1$, from stokes law. Since $6\pi\eta a$ are constants, then this equation can be written as kv_1. This is equal to mg, so $kv_1 = mg....1$, ignoring the up thrust due to air

Suppose the drop now gains charge *q* and an electric field *E* is applied to oppose gravity and slow the falling of the drop. The drop then has a smaller terminal velocity v_2 .Since the force on the drop due to *E* is *qE,* then, if the mass of the drop has remained constant, then

$mg - qE = kv_2 ....2$, substituting 1 in 2 gives $kv_1 - qE = kv_2 \Rightarrow qE = kv_1 - kv_2 = k(v_1 - v_2)$

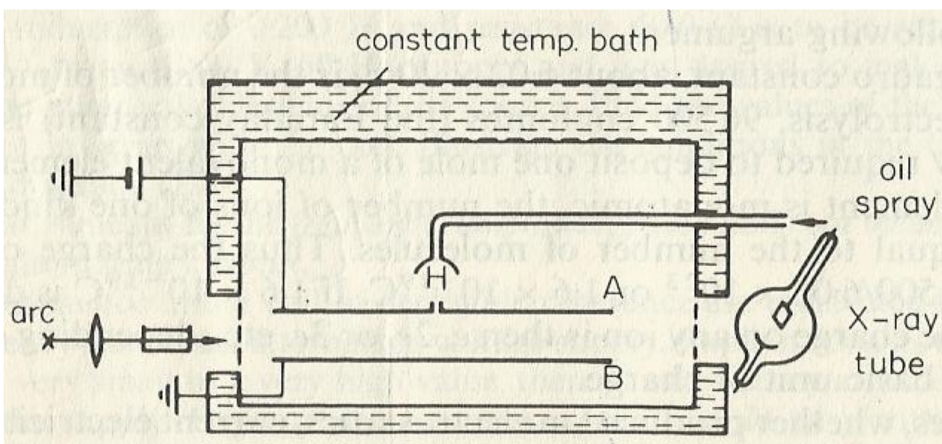
$$\Rightarrow q = \frac{k}{E}(v_1 - v_2) ....3$$

The weight of the oil drop = volume x density x g= $\frac{4}{3}\pi a^3 \rho g$, the up thrust on the drop due to air = weight of air displaced = $\frac{4}{3}\pi a^3 \delta g$

$mg = weight - upthrust$, so $6\pi\eta a v_1 = \frac{4}{3}\pi a^3(\rho-\delta)g$, thus $a = \left(\frac{9\eta v_1}{2(\rho-\delta)g}\right)^{\frac{1}{2}}$

It also follows that $q = \frac{6\pi\eta}{E}\left[\frac{9\eta v_1}{2(\rho-\delta)g}\right]^{\frac{1}{2}}(v_1 - v_2)$

Determination of charge Q of an oil drop using Millikan's experiment



Procedure

i. The fine oil spray atomizes the oil and creates tiny drops above the hole H.

ii. The oil drops are observed in a low power microscope by reflected light when the chamber is illuminated.

iii. With the p.d off, the drop is seen as a pin point of light and its down ward velocity v_1 is measured by timing its fall through a known distance, d, by means of a scale in the eye piece.

iv. The field of intensity $E = \frac{V}{d}$ is applied between the plates of separation d. This is adjusted until the drop remains stationary in the chamber.

v. The quantity of charge is then calculated from $q = \frac{k\, v_1}{E}$, where $k = 6\pi\eta a$, where a is the radius of the drop calculated from $a = \left(\frac{9\eta v_1}{2(\rho - \delta)g}\right)^{\frac{1}{2}}$. Knowing d, the charge q can be obtained.

Assumptions

a) Stokes law holds.

b) Oil drops are identical.

c) The temperature of the apparatus is assumed to be constant.

N.B:

i. Millikan found that the friction between the drops when they are formed by the spray create electric charge, but to give a drop an increased charge, an x- ray tube is operated near the chamber.

ii. Millikan found that, when an oil drop gained charges, the velocity changes $v_1 - v_2$, varied by integral multiples of a basic unit. Thus the charge q is proportional to $(v_1 - v_2)$. Therefore the charge is a multiple of some basic unit.

iii. The oil drop is expected to move downwards even when the electric field is applied due to the influence of its weight.

iv. If the applied p.d between the plates is high enough, then upward motion of the oil drop can occur as the question may require, therefore the equations used above can be adjusted accordingly.

v. Charge $q = ne$, where n is the number of electrons and e is basic charge equal to 1.6×10^{-19}C.

Quantization of charge

Millikan found, working with hundreds of drops, that the charge q was always an integral multiple of basic charge, which is about 1.6×10^{-19}C so he concluded that charge was in form of packets called quanta.

Precautions

i. The experiment is carried out under constant temperature bath to eliminate convectional currents, shield the apparatus from drought thus the drop does not drift sideways and keep viscosity and temperature constant.
ii. The experiment is carried out in still air. A non volatile oil should be used to prevent evaporation which would otherwise alter the mass of the drop.
iii. The oil jet sprays oil in small quantities.
iv. The experiment is carried out under low pressure to reduce errors due to evaporation.
v. Use of non – volatile drop prevents evaporation which would otherwise alter the mass of the drop.

Example

1. Calculate the radius of the oil drop which falls with a terminal velocity of $3.0x10^{-4}ms^{-1}$ through air of viscosity $1.8x10^{-5}Nsm^{-2}$. Assume the density of air is negligible.(density of oil is $900kgm^{-3}$). If the charge on the drop is is -3e, what p.d must be applied between the plates 5mm apart in order to keep the drop stationary. ($e = 1.6x10^{-19}C$)ans: ($1.6x10^{-6}m$,)
2. In Millikan's experiment, the horizontal plates are 1.5cm apart. With the electric field switched off, an oil drop is observed to fall with a steady velocity of $2.5x10^{-2}cms^{-1}$. When the field is switched on, the upper plate being positive, the drop just remains stationary when the p.d between the plates is 1500V.
   i. Calculate the radius of the drop..(density of oil is $900kgm^{-3}$)
   ii. How many electronic charges does it carry?
   iii. If the p.d between the plates remains unchanged, with what velocity will the drop move when it had collected two more electrons. (Viscosity $1.8x10^{-5}Nsm^{-2}$).
3. In the measurement of electron charge by Millikan's apparatus, a potential difference of 1.6Kv is applied between two horizontal plates 14mm apart. With the potential differences switched off, an oil drop is observed to fall with a constant velocity $4.0 \times 10^{-4}ms^{-1}$. When the potential difference is switched on, the drop rises with constant velocity $8.0 \times 10^{-5}ms^{-1}$. If the mass of the oil drop is 1.0×10^{-14} kg, find the number of electron charges on the drop. (Assume air resistance is proportional to the velocity of the oil drop and reflect the up thrust due to air)

4. An oil drop carrying a charge of 3e falls under gravity in air with a velocity of $5.0x10^{-4}ms^{-1}$ between two parallel plates 5mm apart. When a p.d of $4.6x10^{3}V$ is applied between the plates, the drop rises steadily. Assuming that the effect of air buoyancy on the drop is negligible, calculate:

   i. The radius of the drop.

   ii. The velocity with which the drop rises. (density of oil is $900kgm^{-3}$, Viscosity $1.8x10^{-5}Nsm^{-2}$)

5. In the measurement of electron charge by Millikan's apparatus, a potential difference of 1.6Kv is applied between two horizontal plates 14mm apart. With the potential differences switched off, an oil drop is observed to fall with a constant velocity 4.0

x $10^{-4}ms^{-1}$. When the potential difference is switched on, the drop rises with constant velocity 8.0 x $10^{-5}ms^{-1}$.If the mass of the oil drop is 1.0 x 10^{-14} kg, find the number of electron charges on the drop. (Assume air resistance is proportional to the velocity of the oil drop and reflect the up thrust due to air)

6. An oil drop of mass m having charge q is present in air between two horizontal parallel plates across which an electric field E may be applied. In the absence of E the drop travels a distance x downwards in a time t_1. In presence of E, the drop travels upwards in time t_2.if an oil drop travelling with velocity V is subjected to a resistive force KV, where K is a constant and assuming density of air is negligible, show that $q = \frac{mg(t_1 + t_2)}{Et_2}$.

7. A drop charged q acted on by a vertical electric field acquires a constant velocity *v* in an upward direction when the value of the electric field is E_1, a down ward velocity of the same magnitude when the intensity isE_2, and a down ward velocity 2*v* when no field acts. Assuming the force resisting the moving drop is proportional to its velocity, find the ratio of E_1 to E_2.

8. A charged oil drop of mass 3.27x10^{-15}Kg is held stationary between two horizontal metal plates across which a p.d of 1.0x10^{3}V is applied. If the separation of the plates is 1.5cm. Find the number of electrons on the drop. (n = 3)

9. The electron is stated to have a mass of approximately 10^{-27} g and a negative charge of approximately l.6xl0^{-19} C. Outline the experimental evidence for this statement. Formulae may be quoted without proof. You are not required to justify the actual numerical values quoted.

10. An oil drop of mass 3.25 x 10^{-12} g falls vertically, with uniform velocity, through the air between vertical parallel plates which are 2 cm apart. When a p.d of 1000 V is applied to the plates the drop moves towards the negatively charged plate, its path being inclined at 45° to the vertical.

    i) Explain why the vertical component of its velocity remains unchanged and calculate the charge on the drop.

    ii) If the path of the drop suddenly changes to one at 26° 3(f to the vertical, and subsequently to one at 37° to the vertical, what conclusions can be drawn?

11. Give an account of Millikan's experiment for determining the value of the electronic charge e.

12. When an electric field of intensity E_1 is applied between two parallel plates, a charged iol drop moves upwards with a constant velocity V_1. When the direction of this field is reversed, the terminal velocity of the same drop becomes V_2 downwards. The drop can be held stationery between the plates by a field of strength E_2. Assuming the resistive force is directly proportional to the velocity, show that the velocities are related by the equation

$\frac{V_1}{V_2} = \frac{E_1 - E_2}{E_1 - E_2}$.

13. A charged oil drop falls under gravity with a velocity of $4.90 \times 10^{-4} ms^{-1}$ between two parallel plates 5mm apart. When a p.d of 45vvV is connected between the plates, the drop rises steadily with a velocity of $1.10 \times 10^{-4} ms^{-1}$. Assuming that the effect of buoyancy on the drop is negligible, calculate the:
    i. Radius of the drop.
    ii. Number of electrons on the drop. (density of oil = $900 kgm^{-3}$, coefficient of viscosity of air = $1.81 \times 10^{-5} Nm^{-2}s$)

14. Describe an oil drop method of determining the electronic charge, e. How may Avogadro's constant be found when e is known?

15. An oil drop of radius 1.000×10^{-3} cm falls freely in air, midway between two vertical parallel metal plates of large extent, which are 0.5000 cm apart, and its terminal velocity is $1.066 cms^{-1}$. When a potential difference of 3000 volts is applied between the plates, the path of the drop becomes a straight line inclined at an angle of 31° 36' to the vertical. Find the charge on the drop. (Assume the viscosity of air to be $1.816 \times 10^{-5} kg\ ms^{-1}$.)

16. An electron with a velocity of 10^7 m s^{-1} enters a region of uniform magnetic flux density of 0.10 Wb m^{-2}, the angle between the direction of the field and the initial path of the electron being 25°. By resolving the velocity of the electron. Find the axial distance between two turns of the helical path. Assume that the motion occurs in a vacuum and illustrate the path with a diagram, (e/m =1.8×10^{11} coulomb kg^{-1}.)

17. Describe and give the theory of the Millikan oil drop experiment for the determination of the electronic charge. What is the importance of the experiment?

18. In Millikan's experiment a single charged drop was found to fall under gravity at a terminal velocity of 0.0040 cm per second and to rise at 0.0120 cm per second when a field of 2×10^5 volt per m was suitably applied. Calculate the electronic charge given that the radius, a, of the drop was 60×10^{-7} m and that the viscosity of the gas under the conditions of the experiment was 1.80×10^{-5} N s m^{-2}.

19. In a Millikan-type apparatus the horizontal plates are 1.5 cm apart. With the electric field switched off an oil drop is observed to fall with the steady velocity 2.5×10^{-2} cms^{-1}. When the field is switched on the upper plate being positive, the drop just remains stationary when the p.d. between the two plates is 1500 volts.

    (a) Calculate the radius of the drop.

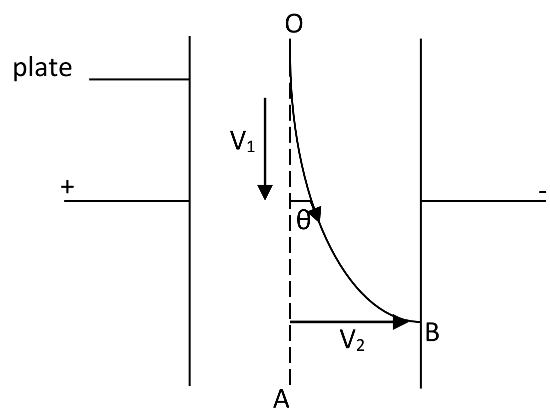
    (b) How many electronic charges does it carry?

(c) If the p.d. between the two plates remains unchanged, with what velocity will the drop move when it has collected two more electrons as a result of exposure to ionizing radiation?

20. Taking this electronic charge to be - 1.60×10^{-19} coulomb, calculate the potential difference in volts necessary to be maintained between two horizontal conducting plates, one 0.50 cm above the other, so that a small oil drop, of mass1.31 x 10^{-11}g with two electrons attached to it, remains in equilibrium between them. Which plate would be at the positive potential?

Deflection of an oil drop between two parallel vertical plates

Consider between two vertical plates at different potentials, one negative and the other connected to the positive terminal as shown below.



With no p.d across the plates, the drop falls vertically down wards with velocity V_1 in the direction OA, thus if m is the mass, then weight of the drop is mg, thus $F = mg = kV_1$........1

When the p.d is switched on, the drop is deflected towards the negative plate at an angle θ to the vertical direction because of the force due to electric current. Therefore, $F_2 = kV_2 = qE$.....2

Dividing 2 by 1 gives, $\frac{qE}{mg} = \frac{kV_2}{kV_1} = \frac{V_2}{V_1} \Rightarrow qE = \frac{V_2}{V_1} mg \quad \Rightarrow q = \frac{V_2}{V_1}\frac{mg}{E}$, but $\tan\theta = \frac{V_2}{V_1}$.

Therefore the charge q is calculated from $q = \frac{mg}{E}\tan\theta$, where θ is the angle that the drop makes with the vertical. $E = \frac{V}{d}$, where V is p.d across the plates and d is plate separation.

Example

1.(a) An oil drop of mass $3.25x10^{-15}$kg falls vertically, with uniform velocity, through the air between parallel plates which are 2cm apart. When a p.d of 1000V is applied to the plates the drop moves towards the negatively charged plate, its path being inclined at 45^0 to the vertical.

i. Explain why the vertical component of its velocity remains unchanged.
ii. Calculate the charge on the drop.

Solution

i. This is because the drops fall steadily due to viscosity since the electric force is horizontal and has no component in the vertical direction.

ii. Using $q = \frac{mg}{E}\tan\theta$, where m = $3.25x10^{-15}$kg, θ = 45^0, g = 9.81, $E = \frac{V}{d} = \frac{1000}{2x10^{-2}} = 5x10^4 Vm^{-1}$ thus $q = \frac{3.25x10^{-15} x9.81\tan 45}{5x10^4} =$

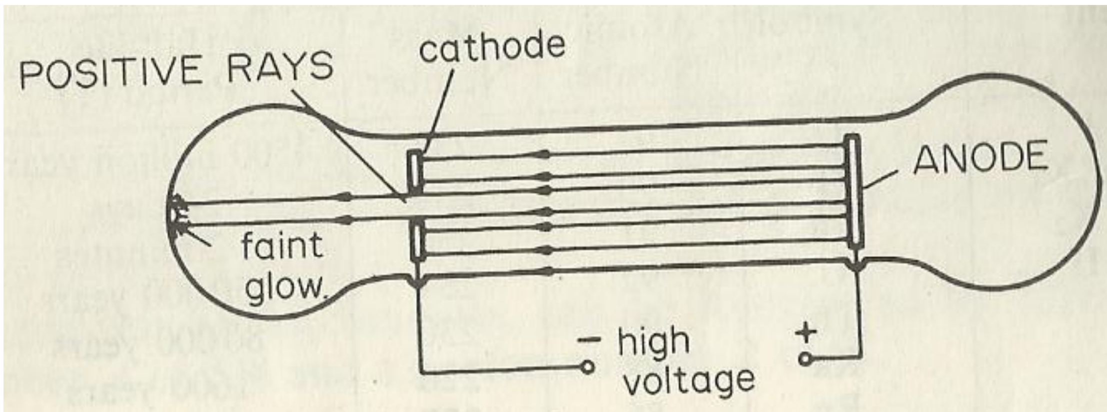
b) If the path of the drop above is suddenly changed to one at $26^0 30^1$ to the vertical, and subsequently to one at 37^0 to the vertical, what conclusions can be drawn?. Ans: the charge on the drop changes from 4e to 2e and then to 3e.

POSITIVE RAYS (IONS)

Production

Positive rays are produced when cathode rays in a discharge tube collide with the gaseous atoms and knock off (strip off) some electrons from the atom. The positive ions formed are accelerated to the cathode and these streams of positive ions constitute positive rays.

X b4



Properties

i. They travel in straight lines.
ii. They are positively charged.
iii. They are more massive compared to cathode rays.
iv. Have smaller specific charge compared to cathode rays because of high mass.
v. They are deflected by strong electric field in opposite direction to that of cathode rays in the same field.

Differences between cathode rays and positive rays

Cathode rays	***Positive rays***
Fast moving electrons	Positive gas ions
Negatively charged	Positively charged
Travel with same velocity	Exhibit arrange of velocities
Not related to gas in a tube	Related to gas in a tube

Light	Heavy
Produce x- rays when they strike a metal	Don't produce X- rays when they strike a metal
Deflected more in electric and magnetic fields	Deflected less in electric/ magnetic field

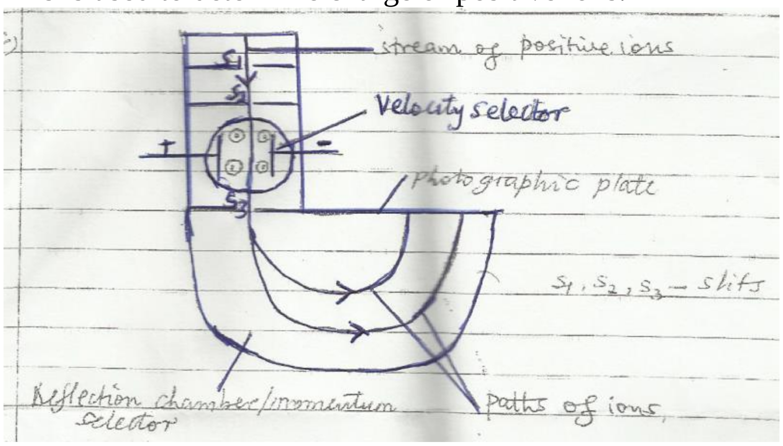
N.B:

a. Avogadro's constant (6.02×10^{23}atoms or mol^{-1}) is the number of atoms in one mole of a substance.

b. Faraday's constant (F = 96500C) is the quantity of charge required to deposit one mole of a mono valent ion during electrolysis.

Bainbridge mass spectrometer

This is used to determine charge of positive ions.



Action

i. A stream of positive ions from the ionizing chamber is directed through slits S_1 and S_2 and emerge as a narrow beam into a region of two crossed uniform electric (E) and magnetic (B).

ii. Only ions with velocity $V = \frac{E}{B}$ will pass through slits S_3 and will enter another region of uniform magnetic field of density of flux density B_1.

iii. Thus the ions describe circular paths of radius r that is recorded.

iv. Magnetic force acting on each ion of charge Q is given by $B_1QV = \frac{mV^2}{r} \Rightarrow B_1Q = \frac{mV}{r}$.

v. Specific charge is calculated from $\frac{Q}{m} = \frac{E}{rB_1B}$.

N.B: If the beam contains atoms of equal mass, all atoms will hit the plate at the same point. If, however, isotopes are present, the heavier atoms will follow a longer radius circle and will hit the plate further to the right.

Measurement of the radius of each isotope's path will allow the determination of its mass.

for selected ions, $v = \frac{E}{B_1}$, $\frac{Q}{m} = \frac{E}{B_1 B_2 r}$. $\Rightarrow \frac{m}{Q} = \frac{B_1 B_2 r}{E}$. Since B_1, B_2 and E are constants, then, $\frac{m}{Q} = kr$, where $k = \frac{B_1 B_2}{E}$ thus $\frac{m}{Q} \alpha r$, for given electric and magnetic fields. The higher the mass of the isotope, the higher the radius of deflection and the smaller the charge. Also since diameter $d = 2r \Rightarrow r = \frac{d}{2}\, hence d_1 = 2r_1 \Rightarrow r_1 = \frac{d_1}{2}$. $d = 2k\frac{m}{Q}$ For two different isotopes, $d_1 = \frac{2m_1}{Q_1}$ and $d_2 = \frac{2m_2}{Q_2}$.

i. $\Rightarrow \frac{d_1}{d_2} = \frac{Q_2 m_1}{Q_1 m_2}$

ii. Separation of isotopes (distance between the images formed) $d_1 - d_2 = \frac{2m_1}{Q_1} - \frac{2m_2}{Q_2}$.

iii. Time taken to strike the photographic plate $t = \frac{d}{v}$, hence $d = tv = 2k\frac{m}{Q} \Rightarrow t = \frac{2km}{vQ}$.

iv. The ratio of the times taken to strike the photographic plate $\frac{t_1}{t_2} = \frac{2km_1}{vQ_1} \div \frac{2km_2}{vQ_2}$

$$\Rightarrow \frac{t_1}{t_2} = \frac{m_1 Q_2}{m_2 Q_1} = \frac{d_1}{d_2}$$

From above, this therefore means that, the Bainbridge mass spectrometer can also be used:

a. To determine the specific charge of positive ions / rays.
b. To separate the isotopes of an ion.
c. To determine the atomic masses of positive ions.

It can therefore be concluded that:

a) The small isotopes experience small deflection and large isotopes experience large deflections.
b) The smaller the charge, the higher the radius of deflection and vice versa.

Example:

1. In a mass spectrograph, an ion of mass number 24 and charge +e and an ion Y of mass 22 and charge +2e both enter the magnetic field with the same velocity. The radius of circular path of X is 0.25m. Calculate the radius of the circular path of Y.

Solution

Using $\frac{m}{Q} = kr$

For ion Y, $\frac{m}{Q} = \frac{22}{2e} = kr..........1$

For ion X, $\frac{m}{Q} = \frac{24}{e} = k \times 0.25 = 0.25k.........2$

Equation 1 divide by 2, gives, $\frac{kr}{0.25k} = \frac{22}{2e} \times \frac{e}{24} \Rightarrow \frac{r}{0.25} = \frac{11}{24}$, hence

$$r = \frac{11}{24} \times 0.25 \Rightarrow r = 0.11m$$

2. A beam of singly ionized atom passes un deflected through a region of crossed magnetic and electric field of 0.10T and $1.0x10^4NC^{-1}$ respectively. When it enters a region of uniform magnetic field, it is deflected through an arc of radius 0.75m. Calculate the magnetic flux density of this magnetic field. (*mass of carbon atom = $2.0x10^{-26}kg$*)

Solution

$$V = \frac{E}{B_1}, \text{ thus } BQV = \frac{mV^2}{r} \Longrightarrow \frac{Q}{m} = \frac{E}{BB_1},$$

$$thus\ B = \frac{mE}{QrB_1} = \frac{2.0 \times 10^{-26} \times 1.0 \times 10^4}{1.6 \times 10^{-19} \times 0.75x.1} = 0.0167T$$

3. The flux density in both electric and magnetic field are $2x10^4Vm^{-1}$ and 0.4T.

i. What is the velocity of an ion which goes undeviated through the slit system?

ii. The source is set to produce singly magnesium charged ions of magnesium isotopes as Mg – 24 and Mg – 26. Find the distance between the images formed by them on the photo graphic plate assuming that the atomic masses of these two isotopes are equal to their mass numbers numerically. (1u = masss of proton = $1.67x10^{-27}$kg)

iii. Calculate the ratios of the times the two isotopes took to complete a semi circle.

Solution

i. For crossed fields in the velocity selector region, $BQv = QE \Rightarrow v = \frac{E}{B} = \frac{2x10^4}{0.4} =$ 50000ms^{-1}

ii. Distance between the images formed $d_1 - d_2 = \frac{2m_1}{Q_1} - \frac{2m_2}{Q_2}$

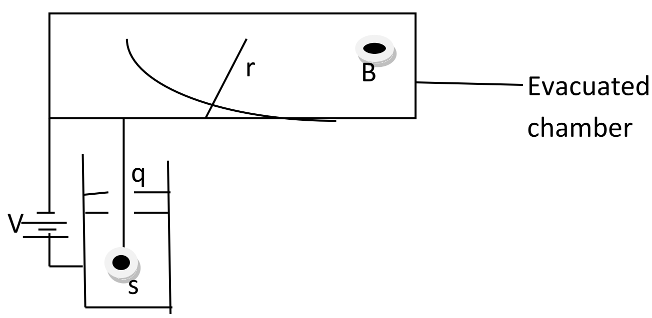
Since the source produces singly charged ions of magnesium, the charge of the isotopes is the same equal to +e, thus $d_1 - d_2 = \frac{2m_1}{Q_1} - \frac{2m_2}{Q_2} = \frac{2 \times 26x1.67x10^{-27}}{1.6x10^{-19}} - \frac{2 \times 24x1.67x10^{-27}}{1.6x10^{-19}} =$

iii. Ratio of the times $\Rightarrow \frac{t_1}{t_2} = \frac{m_1Q_2}{m_2Q_1} = \frac{26 \times 1.67x10^{-27}}{24 \times 1.67x10^{-27}} = \frac{13}{12}$

$t_1 : t_2 = 13 : 12$

Exercise

1. In a Bain bridge mass spectrometer, the magnesium ions $^{+}$Mg-24 and $^{2+}$Mg-26 are deflected in a circular paths by uniform magnetic field. Calculate the:
   a) Ratio of the specific charges of the two ions.
   b) Radius of the path of the heavier ion if that of the lighter ion is 0.36m.
2. Ionized atoms of chlorine-35 and chlorine-37 are passed into the deflection chamber of a mass spectrometer with the same velocity of $2.0x10^4ms^{-1}$. If the flux density of the deflecting chamber is 0.32T., calculate the difference in the radii of the paths of the ions.
3. A uniform flux density B is applied to an evacuated chamber in the plane as shown below



   a) Verify that the path of the ion is circular. ($r = \frac{mV}{Bq}$)
   b) What is the kinetic energy of the ion as it enters the chamber in terms of q and v. ($k.e = qV$)
   c) Show that the mass of the ion is given by the relation$m = \frac{B^2q^2r^2}{2V}$.
4. What velocity will singly charged ions of mass $2.6x10^{-26}$kg have if they travel straight through a velocity selector moving perpendicularly to an electric field of intensity $4kVm^{-1}$ and a magnetic field of flux density 0.10T.
   If these ions then enter a region where there is a magnetic flux density of 0.05T, so that the ions follow a semicircular path perpendicular to the magnetic field direction, what is the radius of the path? ($4x10^4$,0.13)
5. A singly ionized positive atom pass un deviated through a crossed magnetic and electric fields of magnitude 0.3T and $3.6x10^4Vm^{-1}$ respectively. It then enters a region of uniform magnetic field of 0.5T. If the ion describes a circular path of radius 3.0cm, find it's mass. ($2.0x10^{-26}$kg)

QUANTUM THEORY

Terms used

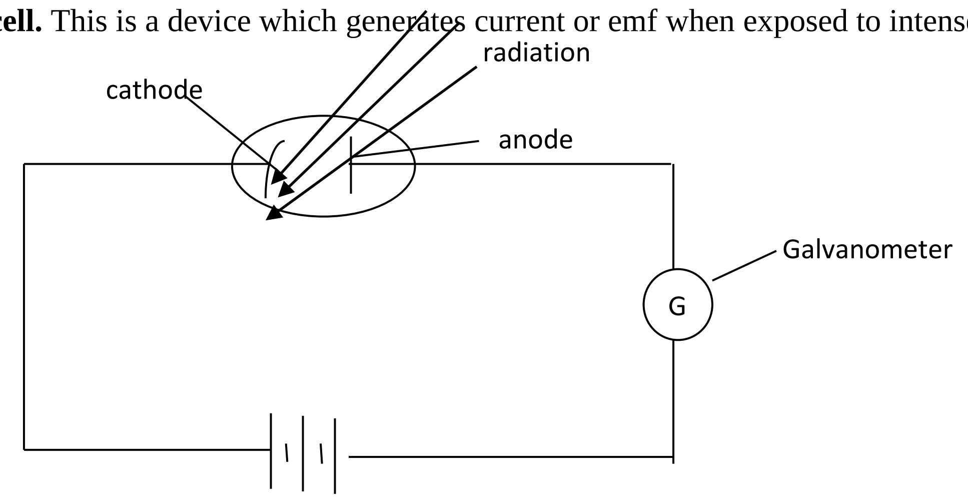
i. ***Photo electric effect*** is the emission of electrons from a metal surface when light of high enough frequency falls on it.
ii. ***Threshold frequency*** is the minimum frequency of the incident radiation required to emit electrons from a metal surface or
The minimum frequency of incident radiation below which no photo electrons are emitted.
iii. ***Threshold wave length*** is the maximum wave length of incident radiation above which no electrons are emitted.
iv. **Stopping potential** is the potential difference required to stop the most energetic electrons from reaching the anode and thus zero photo current.
v. **Photon or quantum** is a tiny packet of electromagnetic energy carried by an atom.
vi. **Work function:** Is the minimum amount of energy required by an electron in order for it to be ejected from the metal surface. The symbol is W_0.

Mechanism

Metals contain free electrons in their lattice. When radiation with sufficient energy falls on the metal surface, electrons acquire the energy to enable them escape from the attractive force of positive ions in the nucleus. These electrons are called photo electrons which give rise to photo current.

Applications of photo electric effect

Photo cell. This is a device which generates current or emf when exposed to intense radiations of light.



Action

When monochromatic light is directed onto a photo sensitive surface of the cathode, electrons are emitted from the cathode and accelerated towards the anode which is kept at a high positive potential with respect to the cathode. As a result, the galvanometer shows a deflection indicating a flow of current due to drift of electrons. If the electrons are kept at a high potential with respect to the cathode, the emitted electrons will experience a retarding potential and will crowd around the cathode thus the galvanometer will show no deflection.

N.B:

When the device is connected to a suitable relay circuit, it can be used to open doors, act as a burglar alarm or as switching device.

TYPES OF PHOTOCELLS

(a) Photo emissive cells
(b) Photovoltaic cells
(c) Photoconductive cells.

Other applications of photo cell include:

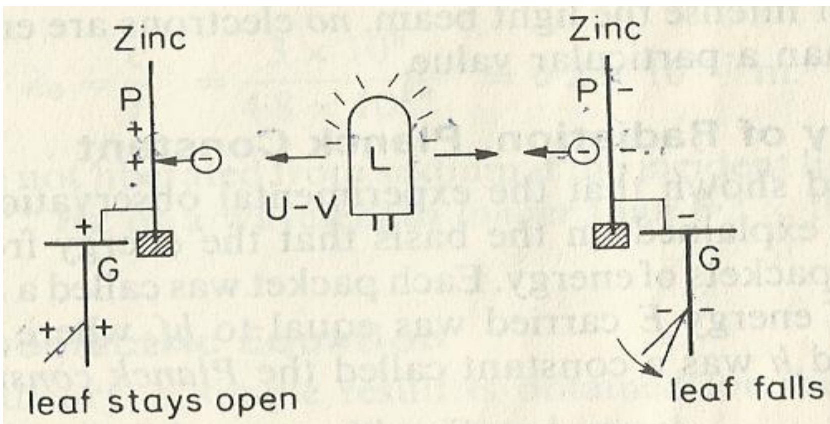
i. Photo voltaic cells. It generates an e.m.f dependent on the intensity of the incident radiation. Such cells are used in solar panels, calculators and for powering electronic watches.
ii. Photo conductive cells.It consists of a plate of a material called photoconductor, whose resistance decreases when it is illuminated by light or infrared radiation, mounted in an evacuated glass bulb. An applied voltage causes current to flow which increases with the intensity of the radiation due to release of electrons in the photoconductor.
iii. Used as automatic devices for switching on light at when it tries to darken.
iv. Detection of an intruder. The intruder intercepts the infrared beam falling on a photocell, hence cutting off of current. This interruption therefore sets the alarm on.
v. Automatic opening of doors and gates. It is made in such a way that coming close to the door/gate interrupts radiation falling on the photo cell. The photo cell is connected to a rely circuit which in turn opens the door or gate.
vi. Automatic counting machines in industries. The items as they move on the conveyer interrupt the radiation falling on the photo cell which triggers the counter to increment.
vii. Used in robots, used as timers to measure speed of runners during a race.

Laws/ properties/ characteristics of photo electric effect

i. Emission occurs only if the frequency of the incident radiation is above some minimum value called threshold frequency.
ii. There is no time lag between irradiation and emission of electrons.
iii. If the incident radiation is of a single frequency, the number of electrons emitted per second is proportional to the intensity of the radiation.

iv. The emitted electrons have various kinetic energies ranging from zero up to maximum value which has no effect on the intensity of the incident radiation.

Evidence/ demonstration of photo electric effect



The surface of the zinc plate P is rubbed with emery paper until the surface is clean and bright. It is then insulated and then connected to the cap of the gold leaf electroscope G as shown above and given a positive charge by induction. In a dark room, the zinc plate is exposed to ultra violet light from the lamp placed near it.

Observation

The leaf stayed open because the emitted electrons are attracted back to P.

When the experiment was repeated using a negatively charged electroscope, the leaf slowly collapsed because the emitted electrons are repelled away from the plate. So the plate loses negative charge and the leaf slowly falls.

Question

Ultra-violet radiation is incident on a clean zinc plate resting on the cap of a charged G.L.E.

Explain what is observed when:

i) The G.L.E is positively charged

ii) Radio wave s / infrared radiation are used instead of ultra violet radiation.

ANSWER

i) No further divergent of the leaf is observed because the ultra violet radiation eject electrons from the metal surface but the electrons are immediately attracted back hence no loss of charge.

ii) Radio waves have low energy thus are unable to release electrons so there will be no effect on the leaf divergence of the electroscope.

Wave or classical theory

According to the wave theory, radiation energy is uniformly spread over the whole wave front. The electron absorbs only a fraction of the total energy. The theory allows continuous absorption and accumulation of energy by electron. Whatever the frequency of the radiation, the electron should accumulate enough be there. Therefore, it cannot account for threshold frequency. Since there is accumulation of energy, the theory predicts a time lag between irradiation and emission of electrons. However, such time lag is not experimentally observed.

Increasing intensity would mean more energy and hence greater value of maximum kinetic energy of the electrons. However, maximum kinetic energy depends on frequency of radiation and not intensity.

Predictions that come from this classical theory explanations

a. The intensity of the radiations should have a proportional relationship with the resulting maximum kinetic energy.

b. The photo electric effect should occur for any light regardless of the frequency or wave length.

c. There should be a delay on the order of seconds between the radiation's contact with the metal and the initial release of photo electrons.

Failures of the wave / classical theory

1. According to the classical theory, radiant energy is uniformly distributed over the whole wave front which is not in line with the fact that electrons absorb only a fraction of the total energy.
2. It also states that there should be time lag between time of irradiation and emission of electrons which time lag cannot be observed.
3. According to wave theory, increasing intensity would mean more energy hence greater kinetic energy, however, maximum kinetic energy does not depend on the frequency of the incident radiation and not intensity.
4. Existence of threshold frequency cannot be explained / accounted for by the wave theory.

Question

a) Explain the inability of the wave theory to account for photo electric effect.

b) Explain briefly how the kinetic energy of the emitted electrons depends on the frequency of the incident radiation.

c) Explain how the classical wave theory fails to account for the observation that photoelectric emission is an instantaneous process.

Quantum theory of energy (plank's theory)

States that energy is emitted in small definite discrete amounts called ***quanta***. Energy in each quantum depends on the frequency of the radiating source thus $E \alpha f$

When a single quantum (photon) interacts with an electron in the metal surface, it gives all or none of it's energy. This implies that only one electron absorbs the energy of one photon. The number of photoelectron is proportional to the number of incident photons (intensity of radiation) of the photo energy. Part is used to overcome attraction of the electron by the metal surface and the rest appear as kinetic energy of the emitted electron.

Minimum energy required to emit an electron is known as work function.

If the photon energy hf is greater than the work function W_o, the electron is dislodged from the surface. The remaining portion of the energy is available as kinetic energy of the emitted electrons. $hf - W_o = K.E.$ if the photo energy is equal to the work function, electrons will be emitted but with zero kinetic energy and the electrons will crowd around the surface of the metal. If the photon energy hf is less than the work function, no photo emission occurs.

The maximum kinetic energy of photo electrons is given by Einstein's equation

$hf - W_o = K.E.$ Thus $hf = W_o + \frac{1}{2}mv^2{}_{max}$

This is the energy lost by a metal when light falls on it. It is called the quantum theory of radiations.

Einstein's explanation of photo electric effect

According to Einstein, each photo electron delivers a quantum of energy, hf, which is absorbed by an electron on the surface of the metal. Part of this energy is used to free the electron from the surface of the metal and if hf is greater than W_0, then the remainder is then available to the electron as maximum kinetic energy.

$k.e_{max} = hf - W_0 \Rightarrow hf = W_0 + k.e_{max}$. Therefore $hf = W_0 + \frac{1}{2}mv^2{}_{max}$ is called Einstein's photo electric equation.

For electromagnetic radiation, velocity of light c is given by c = fλ where c is the speed of light.

Thus $f = \frac{c}{\lambda}$, thus $E = h\frac{c}{\lambda}$. The shorter the wave length, the higher the energy.

How Einstein's equation accounts for emission of electrons.

The incident radiation is in form of packet of energy called photons. A photon interacts with only one electron and gives up all its energy. If it's energy hf is equal to hf_o, even energy will just be sufficient to overcoming electrostatic nuclear attraction force but the photo electrons will not have kinetic energy. But, if hf is greater than hf_o, the difference becomes the kinetic energy of the electron.

Why intensity is independent of kinetic energy.

From quantum theory, intensity is equal to the energy of one photon multiplied by the number of photons per second. An increase in intensity increases the number of photons. This increases the total energy of the radiation but does not change in the individual photon energy; therefore kinetic energy of photons is independent of intensity.

DIFFERENCES BETWEEN X-RAY PRODUCTION AND PHOTOELECTRIC EFFECT:

PHOTOELECTRIC EFFECT	X - RAYS
• Electromagnetic radiation falls on metal surface and electrons are emitt	• Fast moving electrons hit the metal target and x-rays (electromagnetic radiation) is proc
• Little heat is generate	• A lot of heat is generate

Example

1. Explain why light whose frequency is less than the threshold frequency cannot cause photoelectric emission.

ANSWER:

According to the quantum theory, radiation is absorbed or emitted in discreet packets of energy called quanta with energy hf . Electrons would not absorb energy gradually up to the work function W_0 since photons are indivisible. Therefore a photon of energy less than hf_0 is rejected.

2. Explain why the classical theory (wave theory) of light fails to account for the photoelectric effect (emission).

ANSWER:

(i) Instantaneous emission:

According to the wave theory, the radiant energy is spread over the wave front. Since the amount of energy incident on any electron would be extremely small, some time would elapse before an electron escapes from the metal surface. On contrary no such time lag between the start of radiation and start of emission is observed even when the radiation is weak.

(ii) Variation of K.E:

According to the wave theory, more intensity would mean more energy and hence a greater of maximum kinetic energy. But instead maximum kinetic energy depends on frequency of the incident radiation and not on the intensity.

(iii) Existence of threshold frequency

Wave theory predicts continuous absorption and accumulation of energy. Radiations of high enough intensity should cause emission even when the frequency is below the minimum value. The wave theory failed to account for the existence of the threshold frequency.

3. A $100mW$ beam of light of wave length $4.0\times10^{-7}m$ falls on a caesium surface of a photocell.

(i) How many photons strike the caesium surface per second?

(ii) If 80% of the photons emit photoelectrons. Find the resulting photocurrent.

(iii) Calculate the kinetic energy of each photoelectron if the work function of caesium is $2.15eV$.

SOLUTION:

$(i)\ Power = 100mW = 100mJs^{-1}$

$\lambda = 4.0\times10^{-7}m$

$Energy of\ each\ photon (one\ photon) E = hf$

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{4.0\times10^{-7}} = 4.97\times10^{-19}J$$

$$Number of\ photons n = \frac{Total energy}{Energy of\ one\ photon}$$

$$n = \frac{100\times10^{-3}}{4.97\times19^{-19}} = 2.02\times10^{17}s^{-1}$$

$(ii)\ 80\%\ of\ photons emit\ photoelectrons$

$Number of\ electrons emitted n = 2.02\times10^{17}\times0.8 = 1.616\times10^{17}s^{-1}$

$Photocurrent I = ne$

$I = 1.616\times10^{17}\times1.6\times10^{-19} = 2.57\times10^{-2}A.$

$(iii) K.e_{\max} = hf - W_0$

$$= \frac{hc}{\lambda} - W_0$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{4.0\times10^{-7}} - \left(2.15\times1.6\times10^{-19}\right)$$

$= 4.9725\times10^{-19} - 3.44\times10^{-19}$

$= 1.53\times10^{-19}J$

$= 0.944eV$

EXPERIMENTAL EVIDENCE FOR QUANTAM THEORY:

(i) PHOTOELECTRIC EFFECT:

To liberate an electron from a metal surface, a quantum or packet of energy called the work function which is characteristic of the metal surface has to be supplied i.e.

$hf - W_0 = \frac{1}{2}mv^2$ where W_0 is the work function.

(ii) OPTICAL SPECTRA:

A line in the optical emission spectrum indicates the presence of a particular frequency f of light and is considered to arise from loss of energy which occurs in an excited atom when an electron jumps directly or in steps from a higher energy level E_2 to lower energy level E_1.The frequency of the packet of energy emitted is given by $hf = E_2 - E_1$.

(iii) X-RAY LINE SPECTRA:
Electron transition from one shell to another leads to liberation of energy in packets characteristic of the target atom.

DIFFERENCES BETWEEN CLASSICAL THEORY AND QUANTUM THEORY:

CLASSICAL(WAVE) THEORY	QUANTUM THEORY
• It allows continuous absorption and accumulation of ener	• No continuous absorption allowed. The energy is either absorbed or rejec
• Energy of radiation is evenly distributed over the wave fr	• Energy is radiated, propagated and absorbed in packets (quanta or photo
• What matters is total energy of the incident radiation (bea	• What matters is the energy of individual photo

Example:

1. A metal of work function 3.5eV is radiated with light of unknown frequency. The maximum velocity of the photo electrons is $4.72 \times 10^{6} ms^{-1}$. Calculate the maximum wave length of the incident radiation. h is plank's constant = 6.63×10^{-34}Js. mass of electron = 9.11×10^{-31}kg, 1eV = 1.6×10^{-19}J.
   Solution
2. Light of wave length 6.0×10^{-7}m is incident on a zinc plate. If the electrons are emitted with maximum kinetic energy of 2.2×10^{-20}J, calculate the maximum kinetic energy of an electron emitted from a zinc plate with a wave length of 5.0×10^{-7}m.
3. A monochromatic source of power 5mW emits 8×10^{15} photons per second. This light ejects photoelectrons from the metal surface. If the stopping potential is 2V, calculate the work function of the metal surface.
4. Light of frequency 5.0×10^{14}Hz liberates electrons with energy 2.31×10^{-19}J from a certain metal surface. What is the wave length of ultra violet light which liberates electrons of energy 8.93×10^{-19}J from the same surface.
5. Violet light of wave length 0.42mm is incident on a metal surface of threshold wave length 0.67mm. Find the maximum speed of the emitted electrons.
6. A point source emits monochromatic light of wavelength 4.5×10^{-7}m at a rate of 0.2W, uniformly in all directions. Light from the source falls normally on the cathode of area $3.14 \times 10^{-4} m^2$ and threshold frequency 5.0×10^{14}Hz of a photo cell at a distance of 50cm from the source.

i. How many photons leave the source per second?

ii. Calculate the photo current assuming 20% of the photons hitting the cathode liberate electrons.

Solution

i. power $= \frac{E}{t} = nhf$, but $f = \frac{c}{\lambda}$, thus $0.2 = n\frac{hc}{\lambda}$

ii. area on which photons fall = $4\pi R^2 = 4x3.14x0.5^2 =$

Photons on unit area = $\frac{4.55x10^{17}}{4x3.14x0.5^2} =$

Photons incident on an area = $3.14x10^{-4}\,x\,\frac{4.55x10^{17}}{4x3.14x0.5^2} =$

Photons that liberate electrons = $4.55x10^{13}\,x\,\frac{20}{100} = 9.1x10^{12}$

But photo current $I = ne = 9.1x10^{12}\,x1.6x10^{-19} = 1.456x10^{-6}\,A$

7. (a) (i) State the laws of photo electric emission.

(ii) Write down Einstein's equation for photo electric emission.

(iii) Ultra – violet light of wave length 3.3×10^{-8}m is incident on a metal. Given that the work function of the metal is 3.5eV, calculate the maximum velocity of the liberated electron.

8. In an experiment on photo electricity the maximum kinetic energy of photo electrons was determined for different lengths of the incident radiations. The following results were obtained.

Wave length/nm	Maximum Ke/ev
300	2.03
375	1.20
500	0.036

Use the results to determine:-

(i) The work function for the metal.

(ii) The value of Planks constant.

9. An electromagnetic radiation of frequency $5x10^{15}$Hz is incident on a caesium metal and photoelectrons of maximum kinetic energy of $3.0x10^{-18}$J are emitted. When the same radiation is incident on potassium metal, photoelectrons of maximum kinetic energy $2.9x10^{-18}$J are emitted. Determine the ratio of their work functions.

9. Write down Einstein's equation for photoelectric emission. Explain the meaning of the terms in the equation and discuss its significance. Describe briefly how Einstein's equation may be verified experimentally.

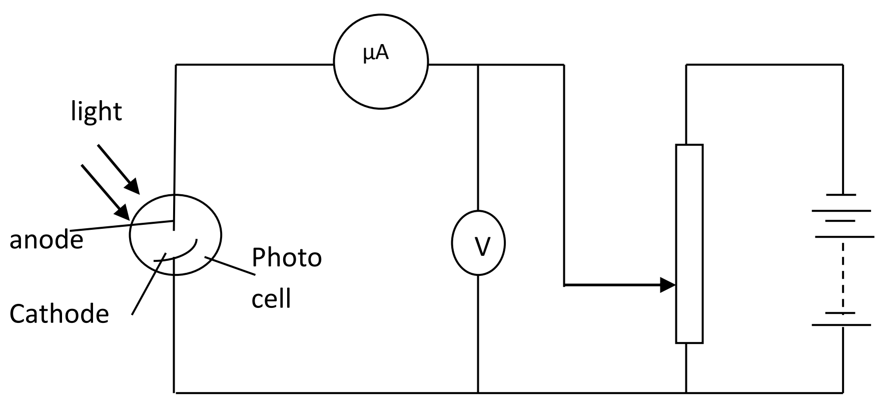
10. An effective point source emits monochromatic light of wavelength 4500 A at a rate of 0.11watt.

(a) How many photons leave the source per second?

(b) Light from the source is emitted uniformly in all directions and falls normally on the cathode of area 280cm^2 of a photocell at a distance of 50 cm from the source. Calculate the photoelectric current, assuming 10 per cent of the photons incident on the cathode liberate electrons. (Planck's constant = 6.6 x 10^{-34}Js; charge of electron = 1.6 x 10^{-19}C.)

11. In photoelectric setup, a point source of light of power 0.0032w emits mono energetic photons. A point source is located at a distance of 0.8m from the center of a stationary metallic sphere of work function 3.0eV and of radius 0.008m. The efficiency of photoelectron emission is one for every 10Exp6 incident photons. Assuming that the sphere is isolated and initially neutral and that photoelectrons are instantly swept away after emission, calculate the number of photoelectrons emitted per second.

Measurement of maximum kinetic energy and stopping potential of a clean metal surface



i. The anode is made negative relative to the cathode.

ii. The cathode is illuminated with monochromatic light so as to emit photo electrons with varying kinetic energy.

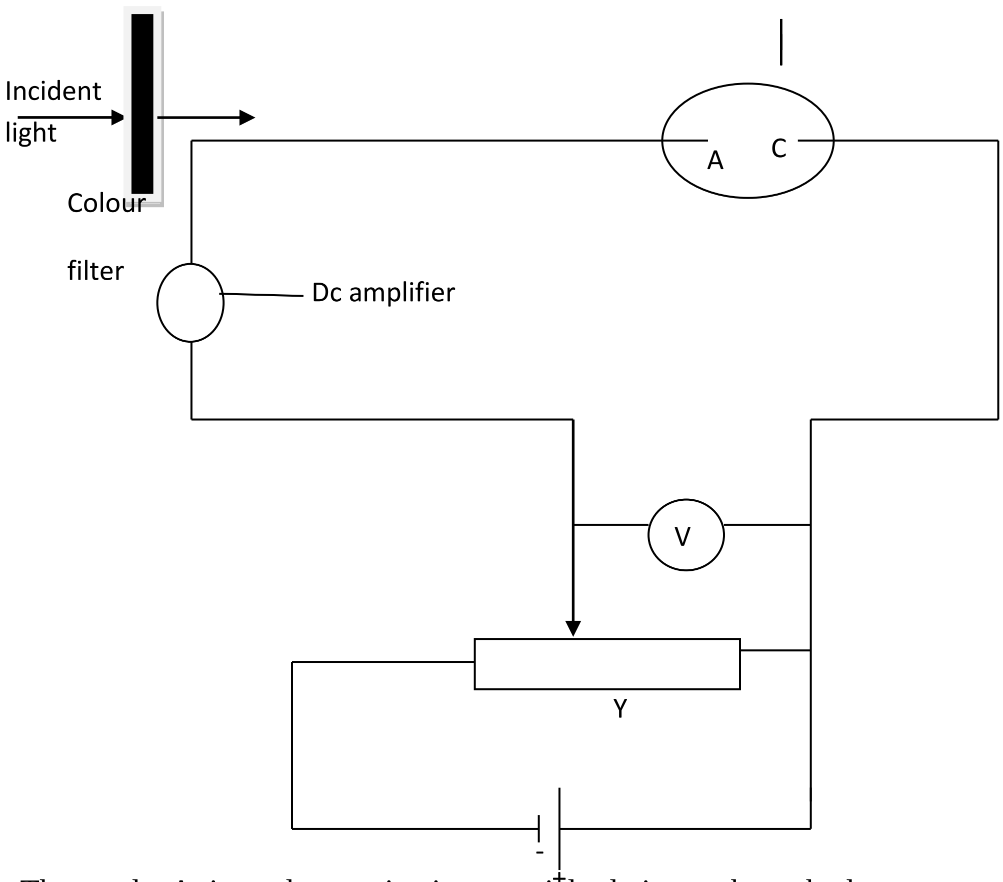
iii. The potential difference is varied until the micro ammeter just reads zero.

iv. The voltmeter reading V_s is noted and this is the stopping potential.

N.B

Since an electron would lose an amount of energy given by charge x p.d moving from C to A, thus $\frac{1}{2}mV^2 = eV_s$. So eV_s is a measure of the maximum energy of the emitted photo electrons.

Measurement of h (Plank's constant)



The anode, A, is made negative in potential relative to the cathode.

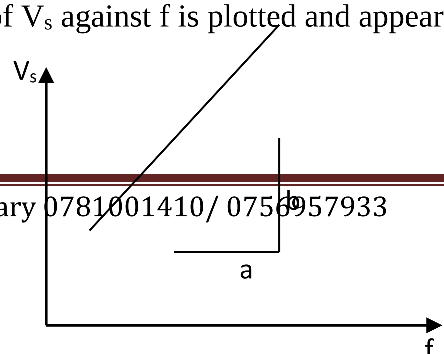
Light of a known frequency, f is directed onto the color filter to reach the photo sensitive cathode C.

The photo electrons emitted from the cathode experience a retarding potential difference (p.d).

The p.d is increased negatively until the current becomes zero and the stopping potential, V_s, is then read from the voltmeter.

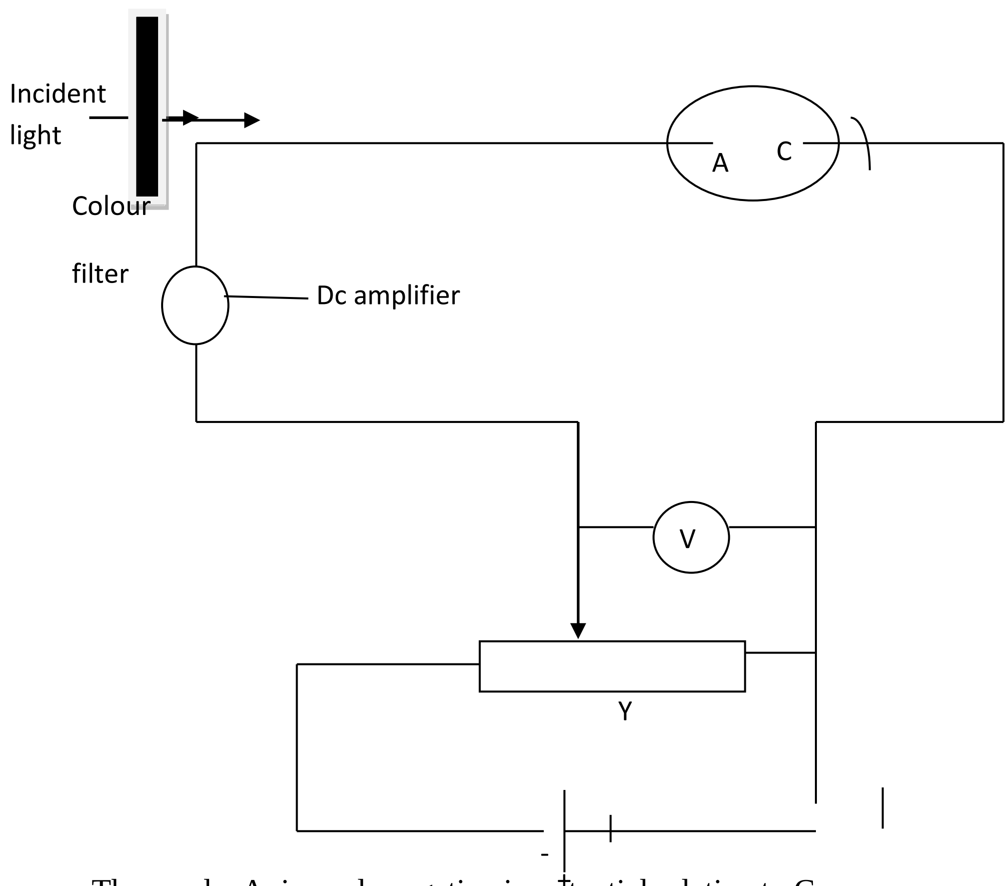
Using pure filters, the frequency of the incident light is varied and the corresponding values of V_s obtained.

A graph of V_s against f is plotted and appears as a straight line graph as shown below.



From Einstein's photo electric equation, $eV_s = hf - W_0 \Rightarrow V_s = \frac{h}{e} f - \frac{W_0}{e}$. thus from Einstein's theory, the gradient $\frac{a}{b}$ is $\frac{h}{e}$. Hence $h = ex\frac{a}{b}$. Knowing, e, the electron charge, h can be calculated.

Verification of Einstein's equation



The anode, A, is made negative in potential relative to C.

Light of a single frequency is directed onto the color filter to reach the photo sensitive cathode C.

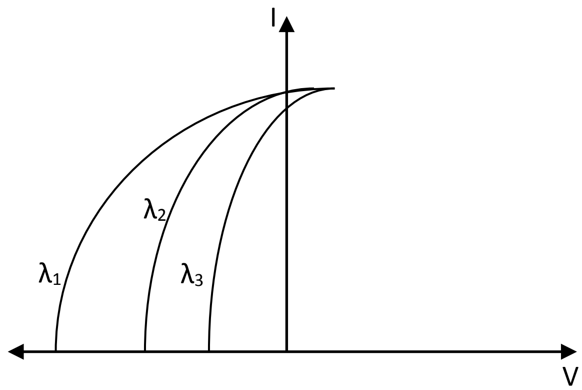
The photo electrons emitted from the cathode experience a retarding potential difference (p.d).

The p.d is increased negatively until the current becomes zero and the stopping potential, V_s, is then read from the voltmeter.

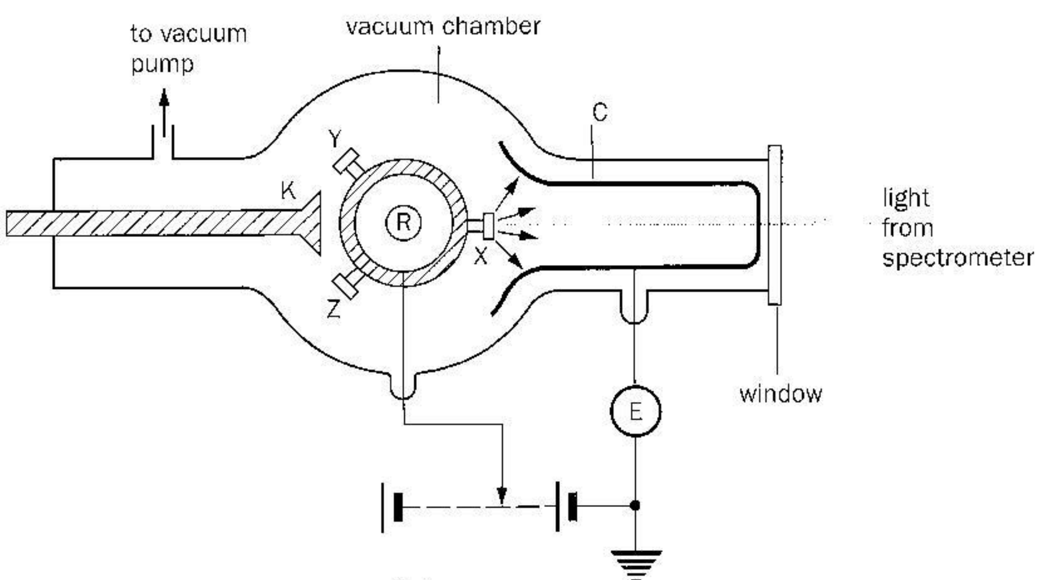
Using pure filters, the frequency of the incident light is varied and the corresponding values of V_s is obtained. A graph of V_s against f is plotted and is a straight line graph verifying Einstein's equation. $eV_s = hf - W_0 \Rightarrow V_s = \frac{h}{e} f - \frac{W_0}{e}$

N.B:

If current I is for different wavelength is plotted against stopping potential, the graph appears as shown below.



Millikan's verification of Einstein's equation and measurement of h



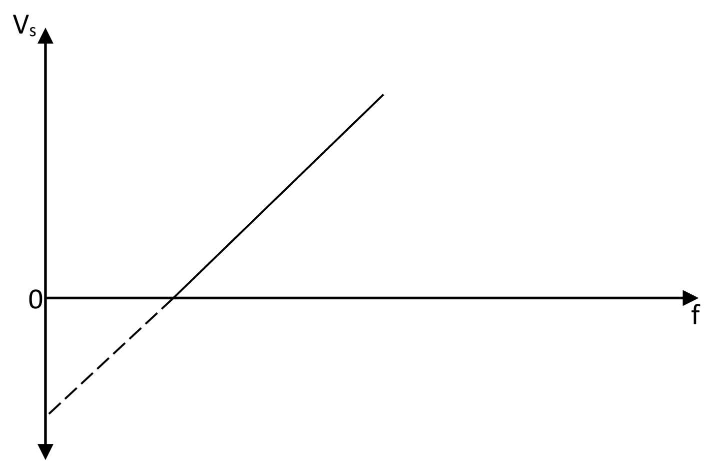
The experimental set up is arranged as shown above.

The alkali metals lithium, sodium and potassium (X,Y and Z respectively) were placed round the wheel W and illuminated with visible light causing them to emit electrons.

The metal A is kept at a variable positive potential by a battery H and illuminated by a beam of mono chromatic light. the photo electrons emitted reach the guaze cylinder G which is connected to an electrometer E whose other terminal is earthed, and a current flows.

The potential V of the collector G is varied positively when all the electrons were collected and then negatively until the stopping potential V_s is obtained.

Light of different frequencies is used and the corresponding stopping potentials obtained. A graph of V_s against f is plotted and appears as a straight line hence verifying Einstein's equation. From Einstein's photo electric equation, $eV_s = hf - W_0 \Rightarrow V_s = \frac{h}{e} f - \frac{W_0}{e}$.



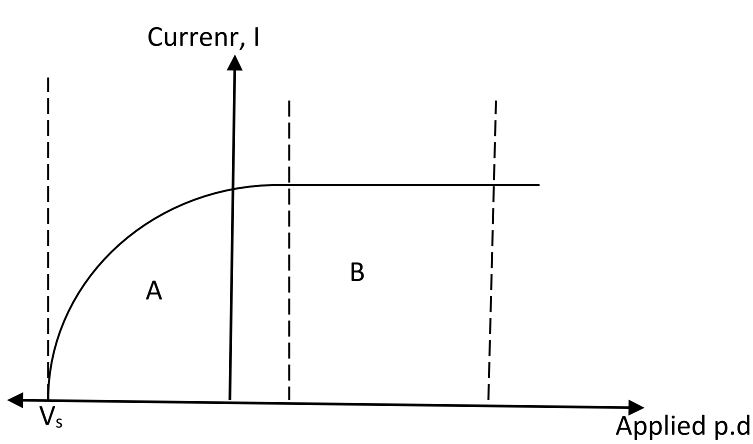
thus from Einstein's theory, the gradient $\frac{a}{b}$ is $\frac{h}{e}$. Hence, $h = ex\frac{a}{b}$. Knowing, e, the electron charge, h can be calculated.

N.B:

i. Changing the target metal gives a graph with the same gradient but different intercepts.
ii. To avoid the formation of oxide films on the metal surface, the metals were evacuated.

iii. The metal surfaces were kept clean by cutting knife K which is turned by the magnet M outside.

Other graphs



Explanation of the regions

Region A

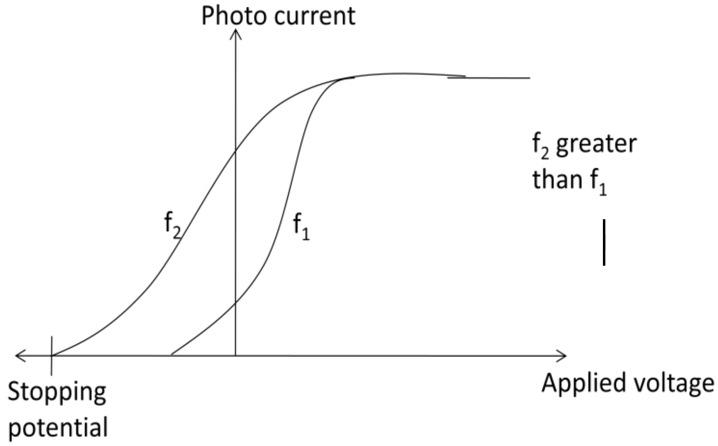
In this region, the ejected electrons move around the photocathode creating an electron cloud which is negatively charged. The number of electrons attracted by the anode is less than the number of electrons emitted. As the applied p.d increases, the space charge reduces and the current increases. The current is said to be space charge limited.

Region B

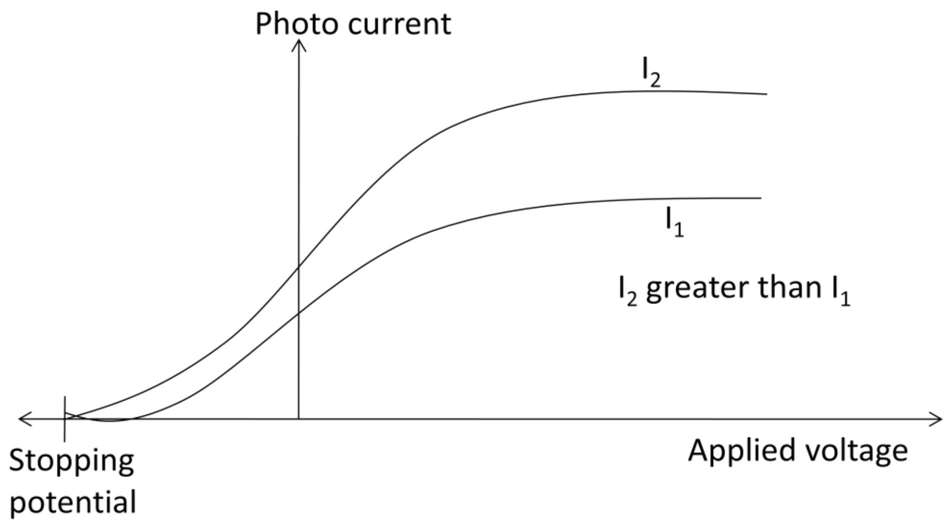
All the ejected electrons are drawn off (attracted) by anode as soon as they are emitted. The space charge is completely removed and current is maximum or saturated. The saturated current is independent of applied p.d but increases with intensity.

Photo current against voltage for two frequencies

(a) Different frequencies of incident radiation at the same intensity

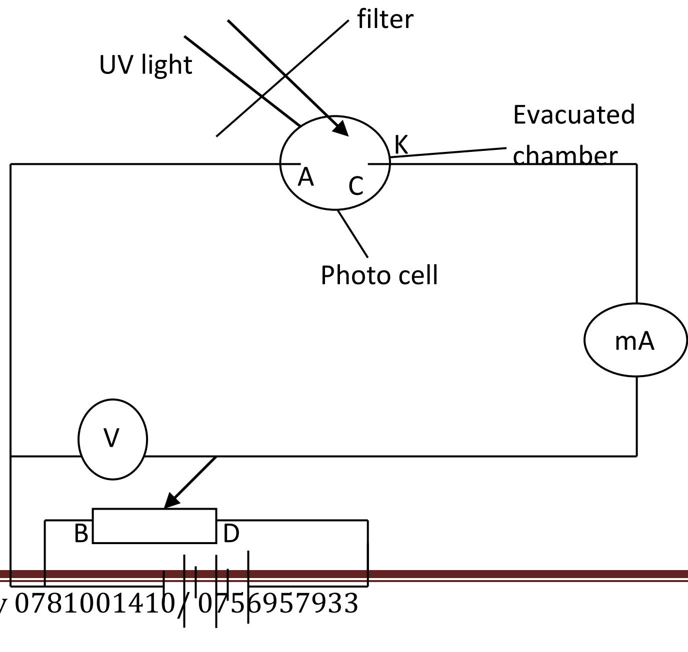


(b) Same frequency but different intensities of incident radiation



Example:

1.



K is a photo cell with A and C made of a metal of work function 0.32eV. The cathode is illuminated with mono chromatic light of wavelength 4.3x10-7m.

a. i) Explain how the current shown by the millimeter will vary when the slider is moved from B to D.

Solution

Current reduces as the slider moves towards D because the cathode is made more positive relative to the anode which reduces the number of photo electrons reaching the anode.

ii) With the slider mid way of BD, explain how the reading of the ammeter would vary when intensity of incident radiation increases.

Solution

Increasing the intensity of the incident radiation increases the negativity of the cathode relative to the anode. This results into increased number of photo electrons thus increased photo current registered by the milli Ammeter.

b) Calculate the voltage reading by the voltmeter at the point when the mA reads 0.0mA.

c) Explain what happens to:

i. Current detected by the milli ammeter when reverse voltage between B and D is increased.
Solution: this makes the anode more negative relative to the cathode which cuts off electrons from reaching the anode thus the milli ammeter shows no current.

ii. Stopping potential if only frequency of the incident radiation is reduced.
Solution: reducing the intensity of the incident radiation reduces the number of photo electrons emitted. Thus less retarding force is needed to reduce photo current to zero thus reducing the stopping potential.

d) What happens when a gas is introduced into the photo cell?

Solution

The fast moving electrons from the cathode will ionize the gas molecules. This will increase the current in the circuit. The current will keep increasing.

2. When mono chromatic light of wave length $5690A^0$ shines on a potassium surface, the photo electric current is stopped by a retarding voltage of 0.2V. When the light of wave length $4050A^0$ is used, the stopping potential is 0.82V. Use the above data to find:

   i. Work function for potassium.

   ii. Value of $\frac{h}{e}$ from this data.($1A^0 = 1x10^{-10}m$)

3. For Caesium the value of it's work function is 1.35 electron-volts,

   (a) What is the longest wavelength that can cause photo-electric emission from a caesium surface?

   (b) What is the minimum velocity with which photoelectrons will be emitted from a caesium surface illuminated with light of wavelength 4000 A?

   (c) What potential difference will just prevent a current passing through a caesium photocell illuminated with light of 4000 A wave length?

Field emission

Is the discharge of electrons from a metal surface subjected to a strong electric field. In absence of a strong electric field, an electron must acquire a certain minimum amount of energy called work function to escape through the surface of a metal which acts as a barrier to escape of electrons.

Secondary emission

Is the release of electrons from a metal surface bombarded by a beam of electrons. Some electrons in the metal surface acquire enough energy to escape the attractive forces of the nucleus by transfer of kinetic energy from bombarding particles.

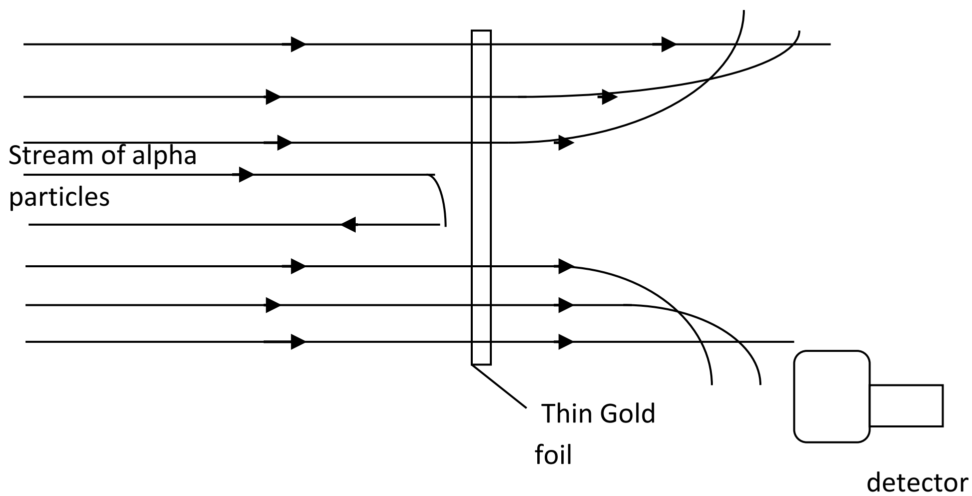
Rutherford's model of an atom

It states that an atom has/ consists of a nucleus, in which all the positive charge and most of its mass is concentrated.

Geiger and Marsden in their experiment on scattering of Alpha particles found that most of the alpha particles passed undeviated through thin foils but some of them were scattered through very large angles. From the results of these experiments, Rutherford proposed Rutherford's model of an atom.

Rutherford's Alpha particle scattering experiment

The beam of alpha particles was directed onto a thin metal (gold) foil and the detector was moved around the foil to detect alpha particles as shown below.



Observation:

i. Most of the alpha particles passed through the gold foil un deviated which implied that most space of the atom is empty.
ii. Few of alpha particles were scattered through small angles. This is because of electro static repulsion between the alpha particles and the positive charge in the centre of an atom.
iii. Very few alpha particles were scattered through large angles. This implies that the positive charge of the atom is concentrated in a small region.
iv. Those particles very close to the nucleus were deflected through large angles.

Rutherford's conclusion

i. Nucleus is positively charged.
ii. All the positive charge and nearly all mass of the atom is concentrated in a very small volume (nucleus) at the centre of the atom.

N.B: when carrying out this experiment**,** a vacuum is necessary to prevent the collision between alpha particles and air molecules so that the alpha particles are not absorbed by air molecules.

Failures of the Rutherford's model

Rutherford's model could not explain the following.

i. Emission spectra. Electrons can jump from one energy level to another of lower energy and the energy difference is emitted as one quantum of frequency f. It could not give any idea about distribution of electrons around the nucleus and its energy.

ii. When an electron is moving in an orbit, it gains acceleration. So an electron moving around nucleus in an orbit is under acceleration. According to Maxwell's electromagnetic theory charged particles when accelerated must emit electromagnetic radiations. So an electron in an orbit will emit radiations. The energy carried from radiations comes from electronic motion. Thus Rutherford's model could not explain about the stability of atom.

Bohr's explanation

From Bohr's model, electron move round the nucleus in circular orbit. While in these orbits, the electron does not radiate energy. Electromagnetic radiation is emitted when the electron makes transition between two orbits. The electrons can only move in a number of fixed orbits in which their angular momentum is equal to $\frac{nh}{2\pi}$, where h is planks constantand n is an integer.

Question: If matter has a wave nature, why is this not observable in our daily experience?

S o l u t i o n

The de Broglie wavelength of macroscopic objects such as a baseball moving with a typical speed such as 30 m/s is very small and impossible to measure. That is, $\lambda = \frac{h}{mv}$, is a very small number for macroscopic objects. We are not able to observe diffraction effects because the wavelength is much smaller than any aperture through which the object could pass.

Question

1. Explain what is observed when a beam of alpha particles is incident on a gold foil.
2. Explain why a vacuum is necessary.

BOHR'S MODEL OF AN ATOM

Bohr atom is an atom with central positive nucleus with electrons revolving round it only in certain allowed orbits and while in these orbits, they do not emit radiations.

BOHR'S POSTULATES: (explanation of the failures of Rutherford's model)

(i) Electrons in atoms exist only in certain discrete orbits and while in these orbits they don't radiate energy.

(ii) Whenever an electron makes a transition from one orbit to another of lower energy, a quantum of electromagnetic radiation is given off. The energy of the quantum of radiation emitted is given by $hf = E_i - E_f$, where E_i is the energy of the electron in the initial orbit, E_f is the energy of the electron in the final orbit, f is the frequency of the radiation emitted and h is Planck's constant.

(iii) The angular momentum of an electron in its orbit in an atom is an integral multiple of $\frac{h}{2\pi}$. Angular momentum of the electron about the nucleus is given by mvr .

Therefore $mvr = \frac{nh}{2\pi}$ ---------------------- (1)

Where n is the quantum number and it takes values $n = 1,2,3---$

Bohr theorem

States that the angular momentum of an electron around the orbit is always a multiple of $\frac{h}{2\pi}$ where h is planks constant. That's to say $mvr = \frac{nh}{2\pi}$

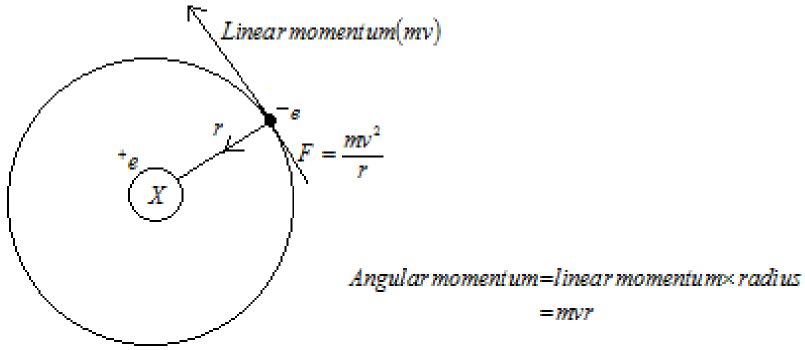
Proof:

Using De Broglie theorem, the motion of an electron around an orbit is wavelike in nature with wave length λ given by $\lambda = \frac{h}{mv}$. If the number of waves in a circular orbit is n, the total distance (nλ)= 2πr $\Rightarrow n\lambda = 2\pi r \Rightarrow \lambda = \frac{2\pi r}{n}$

Therefore, $\frac{2\pi r}{n} = \frac{h}{mv} \Rightarrow mvr = \frac{nh}{2\pi}$ for n = 0, 1, 2... and mvr is angular momentum.

APPLICATION OF BOHR'S POSTULATES TO THE HYDROGEN ATOM

Consider an electron of charge ^{-}e and mass m moving with speed V and acceleration $\frac{V^2}{r}$ in a circular orbit of radius r round a central hydrogen nucleus of charge ^{+}e .



KINETIC ENERGY AND POTENTIAL ENERGY OF ELECTRON IN ORBIT:

An electron has K.E due to its motion round the nucleus and P.E in the electrostatic field of the nucleus.

KINETIC ENERGY OF ELECTRON IN ORBIT

Centripetal force on electron $=\frac{mv^2}{r}$

Electrostatic force on electron $=\frac{ee}{4\pi\in_0 r^2}$

Centripetal force = electrostatic force.

$\frac{mv^2}{r}=\frac{e^2}{4\pi\in_0 r^2}$ ------------------------------------- (2)

But $K.E=\frac{1}{2}mv^2$, from equation (2) $\frac{1}{2}mv^2=\frac{1}{2}\times\frac{e^2}{4\pi\in_0 r}=\frac{e^2}{8\pi\in_0 r}$

$\therefore\ K.E=\frac{e^2}{8\pi\in_0 r}$ ------------------------------------ (3)

POTENTIAL ENERGY OF ELECTRON IN ORBIT:

The potential due to the charge e on the nucleus at a distance r is given by $v=\frac{^+e}{4\pi\in_0 r}$.

P.E of electron $=qv={}^-ev=\frac{^-e.e}{4\pi\in_0 r}=\frac{^-e^2}{4\pi\in_0 r}$ -------------------------------- (4)

The negative sign arise because the nucleus attracts the electron.

Total energy of electron in orbit E = P.E + K.E

$$E = \frac{^{-}e^2}{4\pi \in_0 r} + \frac{e^2}{4\pi \in_0 r}$$

$$E = \frac{^{-}e^2}{8\pi \in_0 r} ------------------------------(5)$$

To find r eliminate V from (1) and (2)

From (1) $V = \frac{nh}{2\pi m r}$

Substituting for V in equation (2) $\quad m\left(\frac{nh}{2\pi m r}\right)^2 \times \frac{1}{r} = \frac{e^2}{4\pi \in_0 r^2}$

$$\frac{mn^2h^2}{4\pi^2 m^2 r^3} = \frac{e^2}{4\pi \in_0 r^2} \quad \Rightarrow r = \frac{n^2h^2 \in_0}{m\pi e^2}$$

$r_n = \frac{n^2h^2 \in_0}{m\pi e^2}$. This gives the radii of the allowed Bohr orbits.

The allowed electron energies are obtained by substituting for r_n

Substituting for r in equation (5) $\quad E_n = \frac{^{-}e^2}{8\pi \in_0 \left(\frac{n^2h^2 \in_0}{m\pi e^2}\right)}$

$\Rightarrow E_n = -\left(\frac{me^4}{8h^2 \in_0^2}\right)\frac{1}{n^2}$. The principal quantum number n takes values $n = 1,2,3----$

NOTE:

(i) The energy of the electron is always negative. This means that the work has to be done to remove the electron from infinity where it is considered to have zero energy. The electron is thus bound to the proton.

(ii) Increasing values of r_n are associated with increasing values of n and therefore with increasing values of E_n. The energy of the stationary state with least energy $(n=1)$ is

$$E_1 = \frac{-me^4}{8 \in_0^2 h^2} = \frac{-9.1\times10^{-31}\left(1.6\times10^{-19}\right)^4}{8\left(8.85\times10^{-12}\right)^2\left(6.625\times10^{-34}\right)^2} J = -2.18\times10^{-18} J = -13.6 eV$$

The energies of the other stationary states can be expressed as $E_n = -\frac{13.6}{n^2} eV$

$$E_2 = -3.39,\ E_3 = -1.59,\ E_4 = -0.85,\ E_5 = -0.54,\ E_6 = -0.38 eV$$

Failures of Bohr Theory

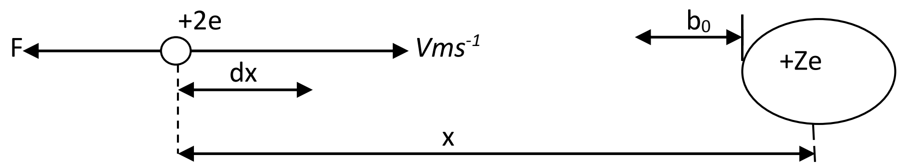
i. It failed to account for the spectra of more complex atoms.
ii. It did not predict the combination of atoms.
iii. It did not explain what is stationary about stationary states.

Question

1. In the simple model of hydrogen, an electron of mass m and charge –e is considered to move in a nearby circular orbit about a proton.
    i. Write down an expression for the force on the electron and show that the kinetic energy of the electron is $\frac{e^2}{8\pi\varepsilon_0 r}$, where r is the radius of the orbit.
    ii. Find the total energy of an electron.
    iii. Given that the angular momentum of an electron is $\frac{nh}{2\pi}$, where n is an integer and h is planks constant, show that the total energy of the electron is given by -$\frac{e^4 m}{8\varepsilon_0^2 n^2 h^2}$.
2. A hydrogen atom is in its first excited state (n = 2). Using the Bohr theory of the atom, calculate (a) the radius of the orbit, (b) the linear momentum of the electron, (c) the angular momentum of the electron, (d) the kinetic energy, (e) the potential energy, and (f) the total energy. ($r_n = n^2 a_0$ yields r_2 = 4(0.0529 nm)

The shortest possible distance of approach b_0 of an alpha particle

Consider an alpha particle projected with the velocity *v* moving towards a positively charged nucleus as shown below,



Repulsive force F $= \frac{2eZe}{4\pi\varepsilon_0 x^2} = \frac{Ze^2}{2\pi\varepsilon_0 x^2}$

For small work done $\Delta w = -F\Delta x = -\frac{Ze^2}{2\pi\varepsilon_0 x^2}\Delta x$, as $\Delta x \to 0$, $\frac{\Delta w}{\Delta x} \to \frac{dw}{dx}$, thus

$$\frac{dw}{dx} = -\frac{Ze^2}{2\pi\varepsilon_0 x^2} \Rightarrow \int \frac{dw}{dx}dx = \int_{\infty}^{b_0} \frac{Ze^2}{2\pi\varepsilon_0 x^2}dx = \left[\frac{Ze^2}{2\pi\varepsilon_0 x}\right]_{\infty}^{b_0} = \frac{ze^2}{2\pi\varepsilon_0 b_0}$$

From the work energy theorem, the work done is equal to the kinetic energy, thus

$$\frac{1}{2}mv^2 = \frac{Ze^2}{2\pi\varepsilon_0 b_0} \Rightarrow b_0 = \frac{Ze^2}{m\pi\varepsilon_0 v^2}$$

Example

1. Show that when an alpha particle collides head on with an atom of atomic number Z, the closet distance of approach to the nucleus χ_0 is given by $\chi_0 = \frac{Ze^2}{\pi\varepsilon_0 mv^2}$, where E is electronic charge, ε_0 is permittivity of free space, m is mass of alpha particle and v is the initial velocity of the alpha particle.
2. A beam of alpha particles of energy 4.2Mev is incident normal to a gold foil. What is the closest distance of approach by the alpha particle to the nucleus of a gold atom? (atomic number of gold = 79)
3. Calculate the least distance of approach of a 3.5Mev alpha particle to the nucleus of a gold atom. (Atomic number of gold = 79). (6.49×10^{-14})
4. Approximately how close can a 5 MeV alpha particle get to a tin nucleus, in a head-on collision if Z = 50.
5. In a Rutherford scattering experiment, an alpha-particle (charge = +2e) heads directly toward a gold nucleus (charge = +79e). The alpha-particle has a kinetic energy of 5.0

MeV. Assuming the gold nucleus to be fixed in space, determine the distance of closest approach.

QUANTIZATION OF ENERGY

Terms used

i. **Ionization energy** is the energy required to remove an electron from an atom and it is completely lost.

ii. **Ionization potential** is the potential required to remove an electron from an atom and it is completely lost.

   Ionization energy is given by $E_\infty - E_0 = eV_I$, where V_I is the ionization potential in volts.

iii. **Excitation energy** is the energy required to move an electron from a lower energy level to a higher energy level.

iv. **Excitation potential** is the potential required to move an electron from a lower energy level to a higher energy level.

   Excitation energy from n = 2 to n = 4 is given by $E_4 - E_2 = eV_e$, where V_e is excitation potential.

v. **Ground state** is the state in which an atom attains its minimum possible energy.

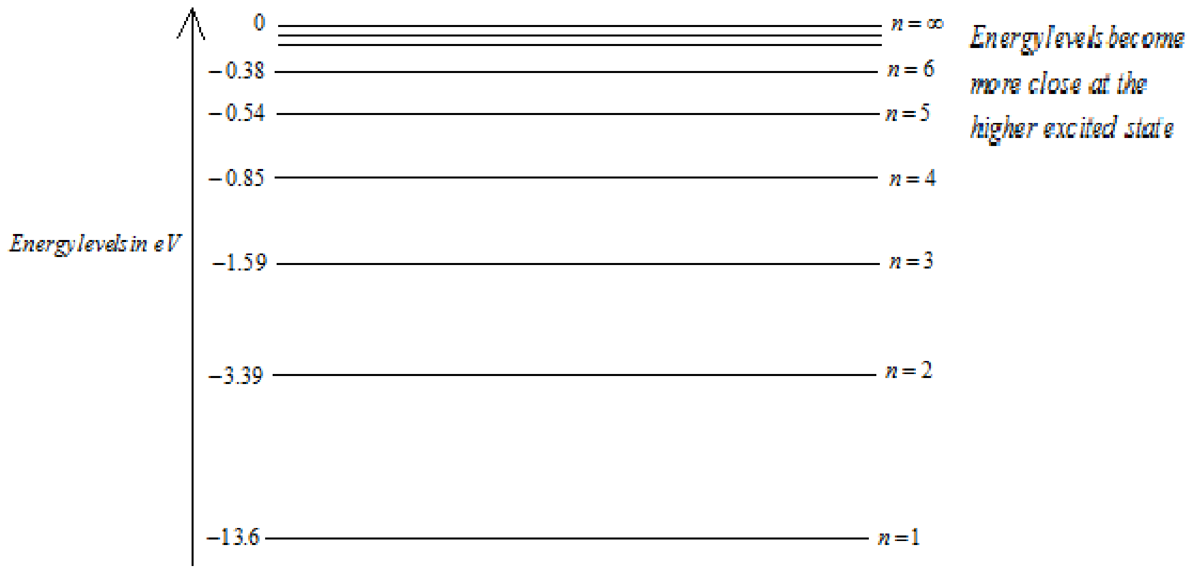
vi. ***Energy levels*** are energies possessed by electrons that correspond to each of the allowed orbits.

vii. ***Electron volt*** is the kinetic energy gained by an electron in being accelerated by a p.d of 1volt.

viii. ***Energy level diagram*** is a diagram in which the energies of different stationary states of an atom are represented by horizontal parallel lines, drawn according to a suitable energy scale.

The energies of the electrons in an atom have only certain values called energy levels of the atom. All atoms of a given element have the same set of energy levels and these are characteristic of the element i.e. they are different from those of other elements. An electron exists in one of a set of well defined energy levels. Energy levels are calculated using Bohr model and expressed in eV. They are usually represented as a series of horizontal lines.

The ground state of an atom is its lowest energy level. Electrons have least energy in the ground state, so this is the case for a normal atom. The atom becomes ionized when an electron receives energy to just exceed its highest energy level and leave the atom. The highest energy level given the value zero and lower energy levels are negative.



The ground state, minimum energy E_1 corresponds to $n = 1$. If E is the energy value of a higher level corresponding to $n = n_2$, then $E - E_1 = \frac{me^4}{8 \in_0^2 h^2}\left(1 - \frac{1}{n_2^2}\right)$.

If an electron moves from a higher energy level to a lower energy level, the difference in energy is released in form of radiation of energy hf , where h is Planck's constant and f is frequency of radiation. So $E - E_1 = hf = \frac{hc}{\lambda} = eV$.

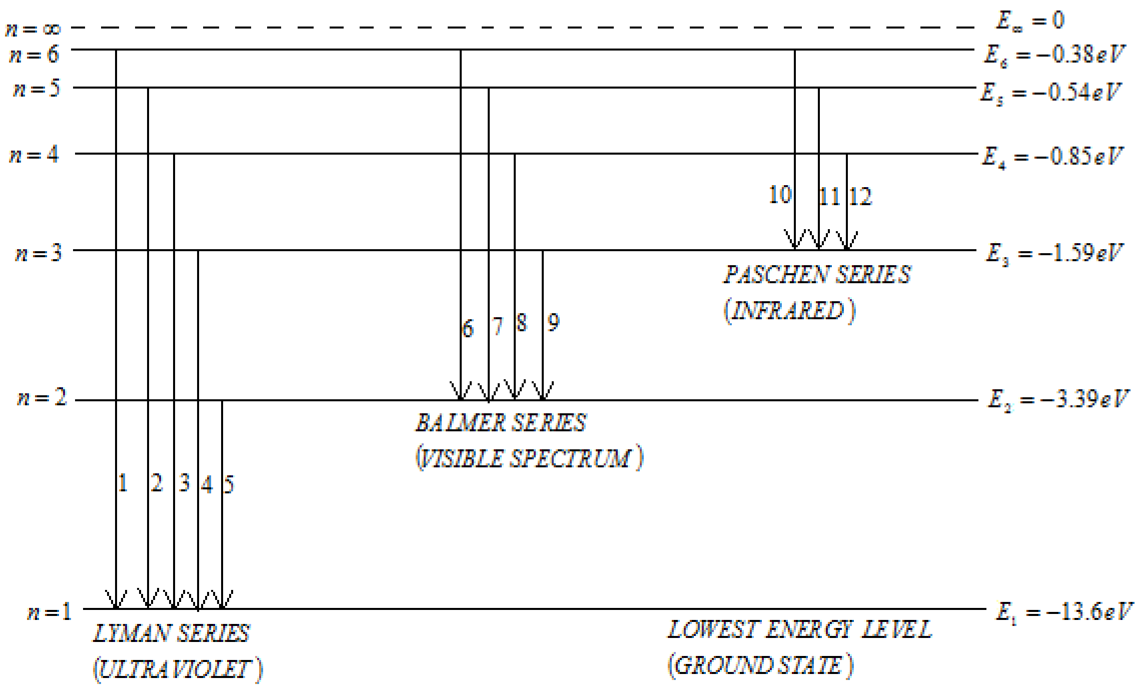
Hydrogen has only one electron and this normally occupies the lowest level and has energy of $-13.6eV$. When the electron is in this level, the atom is said to be in its ground state. If the electron absorbs energy the electron may jump to a higher energy level, the atom is now unstable. It is said to be in excited state. After sometime it may fall back into the lowest level (ground state). The energy originally absorbed is emitted as electromagnetic radiation (wave). An atom absorbs energy by being involved in a collision or by absorbing electromagnetic radiation.

TRANSITIONS:

An electron at changing level E_n if it comes to ground state E_1. It loses energy given by $(E_n - E_1) = hf$. Where h is Planck's constant, f is the frequency of the electromagnetic radiation.

Sometimes an electron may pass through an intermediate state E_m to the ground state. It emits frequencies f_1 and f_2 given by $E_n - E_m = hf_1$ and $E_m - E_1 = hf_2$.

THE ENERGY LEVELS OF THE HYDROGEN ATOM
(SPECTRAL SERIES OF HYDROGEN)



The spectral lines of hydrogen are experimental evidence for the existence of discrete or separate energy levels of the hydrogen atom. Transition involving LYMAN SERIES involve high energy changes. Those in PASCHEN SERIES involve lowest energy changes. The **visiblespectrum** is the BALMER SERIES, the **ultraviolet** is the LYMAN SERIES and the **infrared** is the PASCHEN SERIES.

The lowest energy level or energy state has energy E_1 and is called the **ground state.** The higher state, E_2, E_3, etc are called **excited states.** The quantum of radiation given off in the transition (1) is the most energetic and corresponding wavelength the shortest. The energy required to remove the electron in the ground state to infinity is the ionization energy E_I. Thus $E_I = 0 - E_1$

$$E_I = {}^{-}E_1 = \frac{me^4}{8\epsilon_0^2 h^2} = \frac{9.1\times10^{-31}\left(1.6\times10^{-19}\right)^4}{8\left(8.85\times10^{-12}\right)^2\left(6.625\times10^{-34}\right)^2}\ J \ = 2.18\times10^{-18} J \ = 13.6eV$$

EXAMPLES:

1. Using the above diagram, calculate
   (i) The smallest frequency (longest wavelength) in the Lyman series.
   (ii) The highest frequency (shortest wavelength) in the Paschen series.

SOLUTION:

(i) Smallest frequency in any series corresponds to the change involving the smallest amount of energy. This involves transition from n=2 to n=1.

$$E = hf = \frac{hc}{\lambda}$$

$$E = E_2 - E_1 = \left({}^{-}3.4 - {}^{-}13.6\right)eV = 10.2\,eV$$

$$\lambda = \frac{hc}{E} = \frac{3\times10^{8}\times6.6\times10^{-34}}{10.2\times1.6\times10^{-19}} = 1.2\times10^{-7}\,m$$

(ii) Highest frequency corresponds to the maximum energy change in any series. For Paschen series this is $E_{\infty} - E_3 = (0 - {}^{-}1.5)eV = 1.5eV$

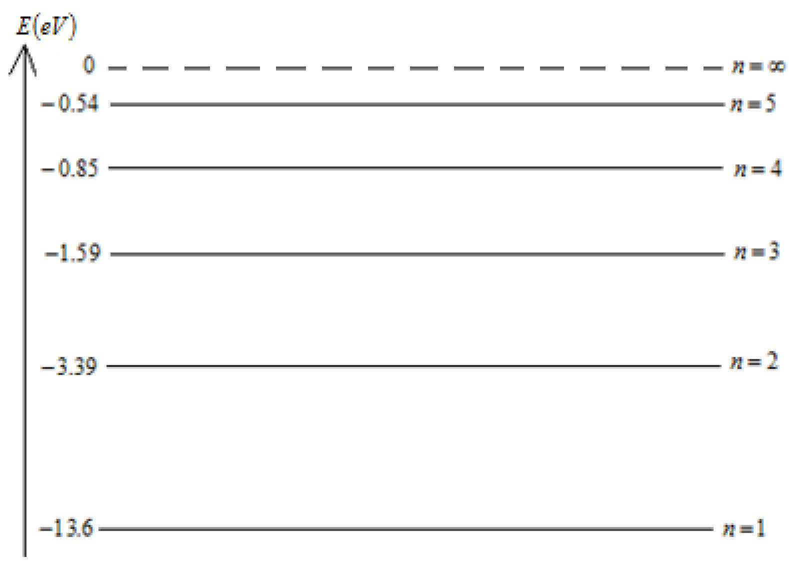
$$E = hf = 1.5\times1.6\times10^{-19}\,J = 2.4\times10^{-19}\,J.$$

2(a) Explain the following.

(i) Bohr atom

(ii) Electron volt

(b) The diagram below shows some energy levels of the hydrogen atom.



(i) Use the diagram to explain the emission spectrum of hydrogen.

(ii) Calculate the speed of an electron which could just ionize the hydrogen atom.

(iii) Calculate the minimum wavelength of the hydrogen spectrum and state the region of the electromagnetic spectrum in which it lies.

SOLUTION:

(a) (i) The Bohr atom is one whose centre, the nucleus, is surrounded by electrons moving in definite circular or elliptical orbits. While the electrons are in these stable orbits, the atom does not radiate. But when an electron makes a transition from an orbit to one of lower energy, electromagnetic radiation is emitted.

(ii) Electron volt is equal to the kinetic energy of an electron which has been accelerated from rest through a p.d of $1V$.

(b) (ii) $\frac{1}{2}mV^2 = -E_1$

$$\frac{1}{2}(9.11\times10^{-31})V^2 = 13.6\times6\times10^{-19}\,J$$

$$V = 2.19\times10^{6}\,ms^{-1}$$

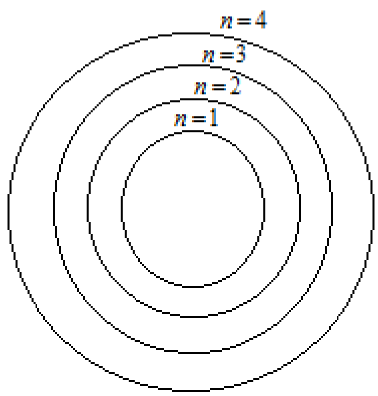
(c) Minimum wavelength corresponds to the transition E_∞ to E_1

$$\frac{hc}{\lambda} = E_\infty - E_1$$

$$\lambda = \frac{hc}{E_\infty - E_1} = \frac{6.625\times10^{-34}\times3\times10^{8}}{13.6\times1.6\times10^{-19}}m = 9.13\times10^{-8}m$$

This lies in the ultraviolet of the electromagnetic spectrum.

3. The diagram below shows the possible electron orbits in the Bohr model for hydrogen. Assume the orbits are circular.



(i) show that the total energy of the atom is $E_n = \frac{-me^4}{8\in_0^2 n^2h^2}$.

(ii) Calculate the wavelength of the radiation that will be emitted when the electron makes a transition from n=4 to n=3 orbits.

SOLUTION:

If the radiation has frequency f and wavelength λ $E_4 - E_3 = hf = \frac{hc}{\lambda}$.

But $E_4 = \frac{-me^4}{8\in_0^2 h^2 4^2}$ and $E_3 = \frac{-me^4}{8\in_0^2 h^2 3^2}$

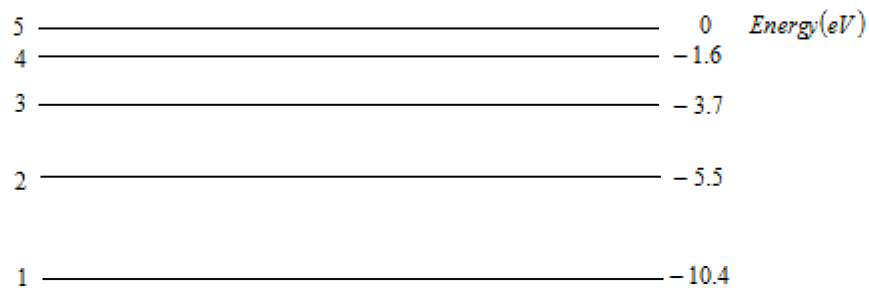
Hence $E_4 - E_3 = \frac{me^4}{8\in_0^2 h^2}\left(\frac{-1}{4^2} - \frac{-1}{3^2}\right) = \frac{hc}{\lambda}$

$$\frac{me^4}{8\epsilon_0^2 h^2}\left(\frac{7}{144}\right)=\frac{hc}{\lambda} \qquad \lambda=\frac{hc\times144\times8\times\epsilon_0^2\ h^2}{7me^4}$$

$$\lambda=\frac{(6.63\times10^{-34})^3\times3.0\times10^8\times144\times8\times(8.85\times10^{-12})^2}{7\times9.1\times10^{-31}\times(1.6\times10^{-19})^4}$$

$$\lambda=1.89\times10^{-6}m$$

4.



Some of the energy levels of mercury are shown in the diagram above. Level 1 is the ground state level occupied by the electrons in an un excited atom.

(i) Calculate the ionization energy of mercury atom in eV (electron volts) and in Joules(J)

(ii) Calculate the wavelength of the radiation emitted when an electron moves from level 4 to level 2.

SOLUTION:

(i) Ionisation energy $E_I=E_\infty-E_1=0-\left(^{-}10.4eV\right)=10.4eV$

But $1eV=1.6\times10^{-19}J$

$E_I=10.4\times1.6\times10^{-19}=1.66\times10^{-18}J$

(ii) $E_4-E_2=hf=\frac{hc}{\lambda}$

$$\lambda=\frac{hc}{E_4-E_2}=\frac{6.63\times10^{-34}\times3\times10^8}{-(1.6-5.5)\times1.6\times10^{-19}}=3.2\times10^{-7}M$$

Wavelength= $3.2\times10^{-7}m.$

An electron cannot occupy any energy level other than those of its own atom for example in an hydrogen atom, an electron cannot occupy an energy level of -0.62eV.

Example

1. The energy levels in mercury atoms are -10.4eV, -5.5eV, -3.7eV and -1.6eV.
   i. Find the ionization energy of mercury in joules.
   ii. Find the ionization potential.

iii. What is likely to happen if a mercury atom in unexcited state is bombarded with an electron of energy 4.0eV, 6.7eV or 11.0eV

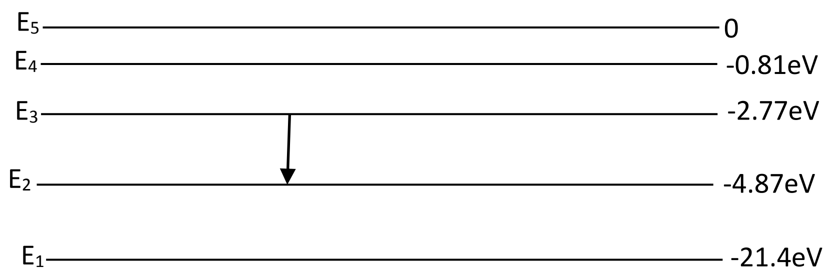
Solution

i. Ionization energy for mercury = +10.4eV, but 1eV = $1.6x10^{-19}$J, thus
= 10.4 x $1.6x10^{-19}$J = $1.664x10^{-18}$J.

ii. Ionization energy =10.4eV = eV_I where V_I is the ionization potential, thus $V_I = 10.4V$

iii. For 4.0eV, 4.0+-10.4 = -6.4eV, for 6.7eV, 6.7+-10.4 = -3.7eV, for 11.0eV, 11.0+-10.4 = 0.6eV.
Therefore, when bombarded with 4.0eV, the electron remains in unexcited state. For 6.7eV, the electron moves to the energy level with energy -3.7eV and when bombarded with 11.0eV, the electron becomes a free electron with kinetic energy of 0.6eV and the atom becomes ionized.

2. The figure below shows some of the energy levels of a neon atom



i. What is the ionization energy of neon?

ii. What is the excitation energy for the transition from energy level E_1 to E_3.

iii. What is the excitation potential for this transition?

iv. In what region of the electromagnetic spectrum does the radiation emitted in transition lie?

Solution

i. $0 - -21.4 = 21.4eV = 21.4 \times 1.6 \times 10^{-19} = 3.424 \times 10^{-18} J$

ii. $E_3 - E_1 = -2.77 - -21.4 = 18.63eV = 18.63 \times 1.6 \times 10^{-19} = 2.98x10^{-18} J$

iii. $eV_e = 18.63eV \Rightarrow V_e = 18.63V$

iv. $E_3 - E_2 = -2.77 - -4.87 = 2.1eV = 2.1x1.6x10^{-19}3.36x10^{-19}J$, but from Planks equation, $E_3 - E_2 = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{3.36x10^{-19}} = \frac{6.63x10^{-34}x3x10^{8}}{3.36x10^{-19}} = 5.9x10^{-7}m$.

Thus Balmer series.

3. The energy levels of a hydrogen atom is given by $E_n = -\frac{21.7\times10^{-19}}{n^2}$ joules where n = 1, 2, 3…

a) Use the result to account for the occurrence of emission and absorption line spectra.

b) Find the shortest wavelength of the radiation which can be emitted by hydrogen atom.

Solution

a) When gases and vapors at low pressure are heated to high temperature, they undergo electronic transitions to high energy levels. This puts the atoms in excited state thus electronic transitions may occur to a vacancy in the lower energy level which causes radiation of definite wavelength to be emitted. So bright lines are formed against dark background. The lines are separated with definite wavelength implying that the energy levels in an tom are quantized. This is *line emission*. An atom can also absorb energy from a photon. If the photon energy E = hf is sufficient to excite an atom to higher energy levels, the photon will be absorbed. When it returns to ground state, excited atom will emit radiations of the same wave length as that of the absorbed photon but equally in all directions. This reduces the intensity of radiation in its direction of incident photon. So dark line is observed on a white back ground. This is the *line absorption spectra.*

b) $E = hf = \frac{hc}{\lambda} = 0 - \frac{21.7\times10^{-19}}{1^2} = 21.7\times10^{-19}$

$$\Rightarrow \lambda = 9.12\times10^{-8}m$$

4. If the atom in ground state absorbs $1.936\times10^{-18}J$ of energy, to what energy level does the electron make transitions?

Solution

Using

Absorbed energy = new energy level – initial energy level

$$= E_n - E_I$$

$$\Longrightarrow E_n = absorbed\ energy + initial\ energy$$

$$\Rightarrow E_n = -13.6eV + \frac{1.93\times10^{-18}}{1.6\times10^{-19}} = -1.5eV$$

$$thus\ in\ the\ energ\square\ leve\square\ n = 3.$$

Emission Spectra

If a gas at low pressure is excited by a high voltage to produce a discharge, and the light is examined in a spectrometer, emission spectra is observed/ seen. This is due to electronic transitions from higher levels to lower levels. This is accompanied by an emission of radiation whose energy is equal to difference in energy of levels involved. A line is therefore formed in the spectrum and appears bright in a dark back ground.

A number of gases such as neon produce line spectrum which consists of well defined lines, each having a particular wavelength of frequency. These separated lines are experimental evidence for the existence of separate or quantized energy levels in the atom.

Types of emission spectra

Emission spectra are classified into:

i. The continuous spectra. Is the one in which all wave lengths are found over a wide range. It is produced in incandescent solids and liquids.
ii. Line spectra. It is obtained from gases eg hydrogen in discharge tube. Also in vaporized sodium salt.
iii. Band spectra. This consists of a series of lines very close together at the sharp edge.

Optical absorption spectra

This is used to determine the elements in the sun.

Procedure

Some elements are vaporized and the vapour irradiated with sun rays.

The sun's spectrum is obtained and dark lines are observed.

The dark lines are characteristic of the elements in the sun thus the elements can be obtained from their absorption spectra.

Question:

1. Explain how the line spectra give an account for the existence of energy levels.
2. Explain the line observation of the of absorption line spectrum nature.

3. a) What is meant by a line spectrum?

   (b) Explain how line spectra for the existence of discrete energy levels in atoms.

4. The energy levels of hydrogen atom are given by the formula $E_n(eV) = \frac{-13.6}{n^2}$, where n=1,2,3……..find the energy levels responsible for the 656nm wave length of hydrogen spectrum.

X- RAYS

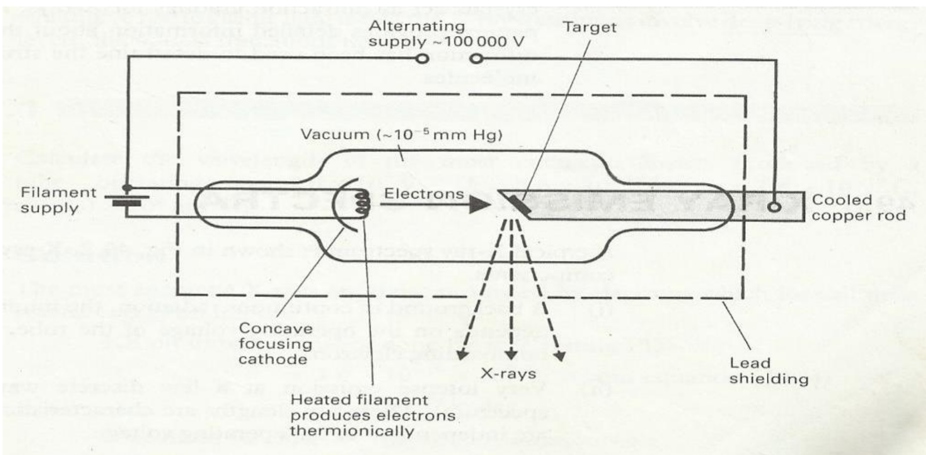
These are electromagnetic radiations of high frequency and short wave length produced when fast moving electrons are stopped by matter.

Production of x- rays

They can be produced using a discharge tube and an x- ray tube

X- ray tube

Structure



Action

The cathode is heated by a low voltage supply and emits electrons by thermionic emission. The emitted electrons are accelerated by extra high tension to very high kinetic energy towards the target. On hitting the metal target, much of their kinetic energy is converted into heat which is carried away by the cooling fins by radiation and convection. The rest is converted to x- rays.

N.B

i. The target is a high melting point metal such as tungsten embedded in a copper rod so as to conduct heat away from the target.
ii. High energy x- rays are called hard x- rays and have a high penetrating power. Low energy x- rays are called soft x- rays and have a low penetrating power.
iii. During production of x- rays, only 1% of the energy of electrons is converted into x- rays. The rest is dissipated as heat. Thus the metal target is made of a metal of high melting point. The metal anode must be cooled using circulation water or oil through cooling fins.

iv. The quality (penetrating power) is increased by increasing the anode potential (p.d across the tube) such that the incident electrons are accelerated by higher p.d hence moving with higher kinetic energy.

v. The evacuation of the tube helps to prevent collision and ionization of gas due to fast moving electrons, which could to reduction in the kinetic energy of the striking electrons.

Effect of intensity on:

(i) Low p.d: increase in low p.d increases the filament current. This increases the temperature of the filament thus more electrons are emitted thus increasing the intensity of x rays.

(ii) Extra high p.d: increase in high p.d increases the speed and kinetic energy of the electrons striking the target. This increases the number of electrons striking the target thus increase in the intensity.

Energy changes in an x- ray tube

Electrical energy in the low voltage supply is converted to heat in the filament to kinetic energy of the electrons to heat in the target to x- rays.

Properties of x-rays

i. X- rays are invisible electromagnetic radiations of short wave length.
ii. X- rays can be diffracted
iii. X- rays can be reflected.
iv. They travel at a speed of light.
v. They are not affected by either electric or magnetic fields.
vi. They ionize materials in which they pass and cause fluorescence.

Uses of x- rays

a. Medical uses

x- Ray photo graphs are used to:

i. Detect fracture in bones, foreign bodies using soft X-rays eg bullets where they penetrate the flesh and absorbed by bones and objects.
ii. Detect lung cancer and tuberculosis using soft X- rays
iii. Treat cancer using hard x- rays which destroy these cancer cells (Radiation therapy uses high-energy radiation to kill cancer cells by damaging their DNA. However, the treatment can damage normal cells as well as cancer cells.
iv. Test the density of bones.

b. Industrial uses

i. Hard x- rays are used to find the faults deeper in any structure eg pipes, machine parts etc. Due to their ability to penetrate certain materials, X-rays are used for a number of nondestructive evaluation and testing (NDE/NDT) applications, particularly for identifying flaws or cracks in structural components. Radiation is directed through a part and onto [a] film or other detector. The resulting shadowgraph shows the internal features" and whether the part is sound.

ii. Used to see characteristic qualities of gemstones and precious metals eg gold, silver.

iii. Used to test the percentage of certain elements in the ore before the ore is taken for commercial mining.

c. Scientific uses

i. Used to study the occurrence of chemical reactions.

ii. X- rays undergo diffraction which is useful in the study of nature of crystals and arrangement of atoms in the crystal.

d. **X-ray crystallography**

Used to determine inter – atomic spacing in the crystal

HOW AN X-RAY IS USED TO LOCATE BROKEN PARTS OF A BONE

Bones are composed of much denser material than flesh hence if x- rays are passed through the body, they are absorbed by the bones onto a photographic plate which produces a shadow photograph and bones

PRECAUTIONS FOR SAFETY

Avoid unnecessary exposure to x –rays

Keep exposure time as short as possible

The x- ray beam should only be restricted to parts of the body being investigated.

Workers dealing with x-rays should wear shielding jackets with a layer of lead.

\Exposure should be avoided for unborn babies and very young children.

N.B: X- rays are considered to be electromagnetic waves because:

a) Their method of production involves accelerated charged particles.
b) They eject electrons from matter by photoelectric emission.
c) They give a line spectrum.

Detection of X – rays. This is based on the fact that X – rays :

a) Ionize gases.
b) Cause fluorescence of metal surfaces.
c) Affect photographic plate.

Question: explain why x – ray production is considered the reverse of photoelectric effect.

Solution

This is because when X- ray photons strike a metal surface, electrons are emitted therefore all the energy is imparted to the electrons thus photoelectric effect. Also, when the energetic electrons strike the metal surface, X – ray radiation are given off.

Energy of x- ray photon $= hf = h\frac{c}{\lambda} = kineticenergy\left(\frac{1}{2}mV^2\right) = eV_a$, where $V_a$is the accelerating p.d and V is the velocity of the energetic electron

Power ,$IV_a = nhf$, where n is the number of photo electrons.

For the anode to be kept at a steady temperature, rate of production of heat at the target = rate of removal of heat from the target = 99%IV, if 1% of energy is converted to x- rays.

Current $I = ne$.Where e is electronic charge.

Example:

1. An x- ray tube operated at a d.c p.d of 40kV, produces heat at the target at a rate of 720W. assuming 0.5% of the energy of the incident electrons is converted to x- ray radiation, calculate:
   i. Number of electron per second striking the target.
   ii. The kinetic energy.
   iii. Velocity of incident electrons.
2. 18kV accelerating voltage is applied across an x- ray tube. Calculate:
   i. Velocity of the faster electron striking the target.
   ii. The minimum wave length in the continuous spectrum of x- rays.
3. An x- ray tube is operated at anode potential of 10kV and anode current of 15mA. Calculate:
   i. Number of electrons striking the anode per second.
   ii. Rate of production heat at the anode, stating the assumptions made.(assuming 1% of energy is converted into x- rays)
   iii. The frequency of the emitted x- ray photon.
4. An x- ray tube is operated at 50V and 20mA. If 1% of the total energy supplied is emitted as x- ray radiation, calculate the :
   i. Maximum frequency of the emitted radiation.
   ii. Rate at which heat must be removed from the target in order to keep it at a steady temperature.
5. In an X-ray tube, if the energy with which the electrons strike the metal target is increased, the wavelength of the characteristic X-rays do not change. Why?

S o l u t i o n :

The characteristic X-rays originate from transitions within the atom of the target, such as an Lshell electron making a transition to a vacancy in the K-shell. This vacancy in the K-shell is caused when an accelerated electron in the X-ray tube supplies energy the the K-shell electron to eject it from the atom. If the energy of the bombarding electron were to be increased, the K-shell electron would be ejected with more kinetic energy. But the energy difference between the K- and L-shells has not changed, so the emitted X-ray has exactly the same wavelength.

6. Distinguish between hard and soft X- rays and state one application of hard and soft X-rays.
7. Electrons of energy 75KeV are stopped by the target of an X – ray tube. Calculate the minimum wave length of the X- rays produced.
8. A 900W X-ray tube operates at a d.c potential difference of 20KV. If only 1% of the total energy supplied is emitted as X-ray radiations, calculate the:
   i. Number of X-ray photons produced per second.
   ii. Shortest wave length of the most energetic X-rays produced by the tube.`
9. Calculate the maximum frequency of x – rays emitted by an x – ray tube operating a voltage of 34.0Kv.
10. An x-rays tube is operated at 20Kv with an electron current of 16mA in the tube. Estimate the
    (i) Number of electrons hitting the target per second
    (ii) rate of production of heat, assuming 99.5% of the kinetic energy of electron is converted to heat (e = 1.6 x 10^{19}C)
11. Describe the properties of X-rays and compare them with those of ultraviolet radiation. Outline the evidence for the wave nature of X-rays.

12. The energy of an X-ray photon is hv joules where h = 6.63 x 10^{-34}Js and v is the frequency in hertz (cycles per second). X-rays are emitted from a target bombarded by electrons which have been accelerated from rest through 105 V. Calculate the minimum possible wavelength of the X-rays assuming that the corresponding energy is equal to the whole of the kinetic energy of one electron.(Charge of an electron = 1.60 x 10^{-19} C; velocity of electromagnetic waves in vacuum = 3.00 x 10^{8} m s^{-1}.)

4. Describe a modern form of X-ray tube and explain its action.

Outline the evidence for believing (a) that X-rays are an electromagnetic radiation, (b) that wavelengths in the X-ray region are of the order of 10^{3} times those of visible light.

6. Describe the atomic processes in the target of an X-ray tube whereby X-ray line spectra are produced. Determine the ratio of the energy of a photon of X-radiation of wavelength 1 A to that of a photon of visible radiation of wavelength5000 A. Why is the potential difference applied across an X-ray tube very much greater than that applied across a sodium lamp producing visible radiation?^.)

7. Draw a labelled diagram of some form of X-ray tube and of its electrical connetions when in actual use.

Electrons starting from rest and passed through a potential difference of1000 volts are found to acquire a velocity of 1.88 x10^7 ms^{-1}. Calculate the ratio of the charge to the mass of the electron in coulombs per kg.

8. You are provided with a glass tube containing an electrode at each end, an exhaust pump, and a source of high potential. Under what conditions (a) does X-RAYS, PHOTOELECTRICITY, ENERGY LEVELS 1093the gas within the tube become a relatively good conductor, (b) is a beam ofelectrons (cathode rays) produced within the tube?

What modifications must be made in the tube in order that a strong beam of X-rays may be produced? What would happen in each case if the potential applied was increased? *

What experiment would you perform to show the effect of a magnetic field on the conducting particles in (a) and (b) and on a beam of X-rays? State the result you would expect.

9. When light is incident in a metal plate electrons are emitted only when the frequency of the light exceeds a certain value. How has this been explained?

The maximum kinetic energy of the electrons emitted from a metallic surface is 1.6 x10^{-19} joule when the frequency of the incident radiation is 7.5 x10^{14}Hz.Calculate the minimum frequency of radiation for which electrons will be emitted. Assume that Planck's constant = 6.6 x10^{-34}J s.

10. Describe a modern form of X-ray tube and explain briefly the energy changes that take place while it is operating. Calculate the- energy in electron-volts of a quantum of X-radiation of wavelength1.5 A.

An X-ray tube is operated at 8000 volts. Why is there no radiation of shorter wavelength than that calculated above (while there is a great deal of longer wavelength)? When the voltage is increased considerably above this value why does the spectrum of the radiation include a few (but only a few) sharp strong lines, the wavelengths of which depend on the material of the target? (Take e = 1.6 x10^{-19} C; h = 6.5x10^{-34}Js; c = 3 x10^8 ms^{-1}.)

11. How are X-rays produced, and how are their wavelengths determined?

Discuss briefly the origin of the lines in the spectrum produced by an X-ray tube that are characteristic of the target metal.

Give a brief account of Moseley's work and the part it played in establishing the idea of atomic number.

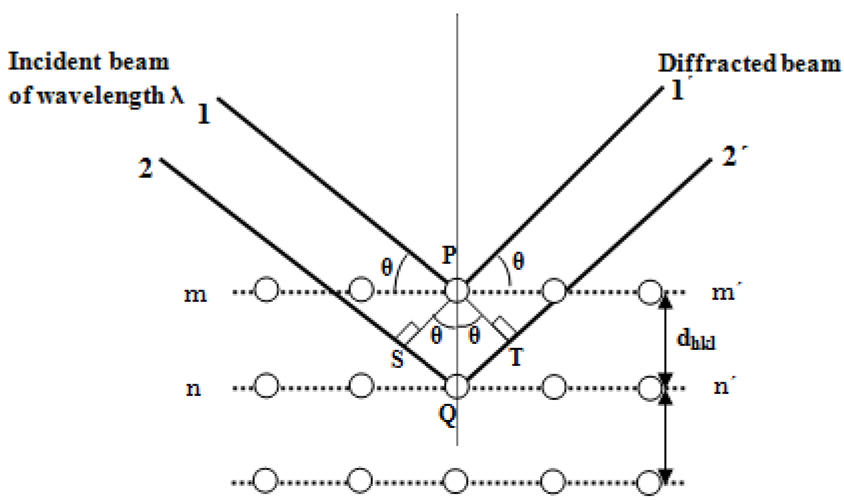
12. Describe and explain one experiment in which light exhibits a wave-like character and one experiment which illustrates the existence of photons.

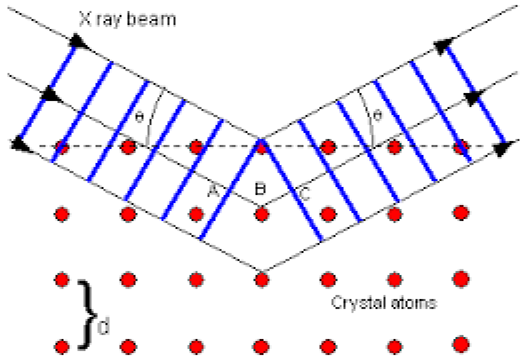
Light of frequency 5.0 x 10^{14} Hz liberates electrons with energy 2.31x10^{-19}joule from a certain metallic surface. What is the wavelength of ultra-violet light which liberates electrons of energy 8.93 x10^{-19} joule from the same surface?(Take the velocity of light to be 3.0x10^{8} ms^{-1}, and Planck's constant (h) to be6.62x10^{-34}Js.)

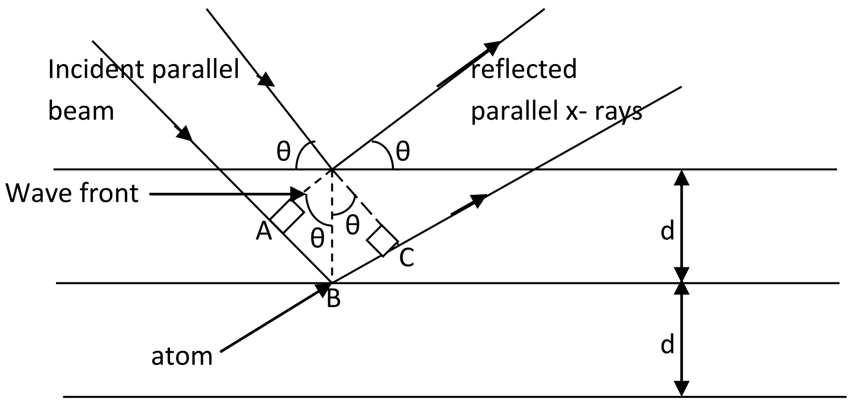
13. What are the chief characteristics of a line spectrum? Explain how line spectra are used in analysis for the identification of elements.

X-ray diffraction

When x- rays move in a single plane of atoms, each atom scatters a small fraction of incident beam. Therefore the atom becomes a source of secondary wave fronts of x- rays. If destructive interference of wave fronts occurs for which the angle of incidence is equal to the angle or reflection, then there is reinforcement of weak reflected beams. The x-rays behave as if they were weakly reflected by a layer of atoms. Other layers for which x- rays penetrate behave as shown below.





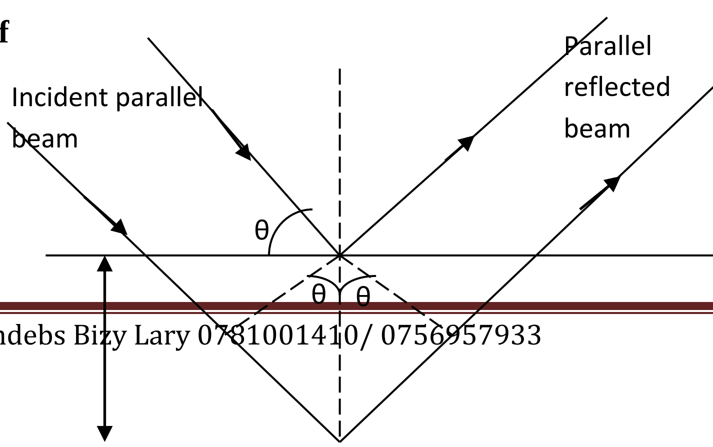


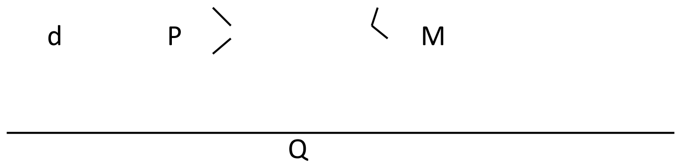
Path difference $= AB + BC$, According to the wave theory of light, for constructive interference of reflected rays, the path difference between two waves is an integral multiple of the wave length of the incident x- ray radiation that's to say nλ, thus if θ is the glancing angle, d is the inter atomic spacing and λ is the wave length, thus an intense x- ray beam is reflected when $2d\sin\theta = n\lambda$ .this is called Bragg's law. Where n is the order of diffraction. n = 1, 2, 3, …

Bragg's law

It states that: when a beam of x- rays is incident on a set of atomic planes, then constructive interference occurs when the path difference is an integral multiple of wave length. Ie 2dsinθ = nλ

Proof





For constructive interference, the path difference between the X – rays reflected from consecutive planes is equal to an integral multiple of the wave length,

Path difference $= AB + BC$, but $AB = BC = d\sin\theta$, thus path difference $= 2d\sin\theta = n\lambda, \Rightarrow 2d\sin\theta = n\lambda$

N.B:

When the crystal is rotated, an intense reflected beam is obtained, thus after several rotations, several orders of diffraction (reflection) may be observed. No images are obtained if λ is greater than 2d.

The intense diffraction (reflection) images from an x- ray tube are due to x- ray lines characteristic of the metal used as the target or anti cathode.

When a monochromatic parallel beam of x- rays is incident on a crystal of atoms, each plane of atoms reflects x- rays of weak intensity. If the reflected beams are in phase with each other, an intense beam is produced. This is constructive interference.

If the angle of incidence is known, then the glancing angle can be obtained.

Applications of x- ray diffraction

i. Measure the average spacing between layers of atoms.
ii. Determine the orientation of a single crystal.
iii. Find the crystal structure of an atom.

Conditions for X – ray diffraction

a) Wave length of the X – rays must be of same order as the interplanar spaces.
b) Parallel beam of x – rays must be incident on planes.

Example:

1. Find the glancing angle for the first order diffraction of x- rays of wave length $2.24\text{x}10^{-10}$m for which the spacing is $4.0\text{x}10^{-10}$m.
2. The spacing between the layers of atoms of a certain crystal is $3.0\text{x}10^{-10}$m. a monochromatic beam of x- rays is found to be strongly diffracted when the glancing angle is $15^{0}42'$. Strong diffraction does not occur for any angle smaller than this. Calculate:
   i. The wave length of the x- rays.

ii. The values of the glancing angle for which strong reflection might occur.

3. Calculate the atomic spacing of sodium chloride if the relative atomic mass of sodium is 23.0 and that of chlorine is 35.5. (density of sodium chloride = $2.18x10^3kgm^{-3}$)

Solution:

Mass of 1 mole of NaCl = 23+35.5 =58.5g = $58.5x10^{-3}$kg

Therefore $6.02x10^{23}$ molecules of NaCl weigh 58.5x10-3kg

1 molecule of NaCl will weigh $\frac{58.5x10^{-3}}{6.02x10^{23}}kg$

But 1 molecule of NaCl contains half Na and half Cl cells

Mass of each cell $\frac{1}{2}x\frac{58.5x10^{-3}}{6.02x10^{23}}kg$

Using volume = $\frac{mass}{density}$, hence

Volume of 1 cell $=\frac{1}{2}x\frac{58.5x10^{-3}}{6.02x10^{23}}x\frac{1}{2.18x10^3}=2.23x10^{-29}$, but volume $=d^3=2.23x10^{-29}\Rightarrow d=2.81x10^{-10}m$

4. A beam of x- rays of wave length $1.0x10^{-10}$m is incident on a set of cubic planes of sodium chloride crystal. Calculate the angle for first order diffraction given that density of sodium chloride is $1980kgm^{-3}$ and molecular mass of sodium chloride is 58.5.

5. A beam of x- rays of wave length $1.0x10^{-10}$m is incident on a set of cubic planes in a sodium chloride crystal. The first order diffraction beam is obtained for a grazing angle of 10.4^0. calculate:

i. Spacing between consecutive planes.

ii. Density of NaCl

Solution:

i. Using Bragg's law

ii. $density=\frac{mass}{volume}$ but volume of NaCl = 2 x volume of each cell = $2d^3$

Mass of 1 molecule of NaCl = $\frac{58.5x10^{-3}}{6.02x10^{23}}=9.73x10^{-26}$

Thus density = $\frac{9.73x10^{-26}}{2d^3}=\frac{9.72x10^{-26}}{2(2.82x10^{-10})^3}=2.17x10^3kgm^{-3}$

6. A monochromatic beam of X- rays incident on a set of planes in a certain crystal. At 0^0C, first order diffraction is observed at a glancing angle of 30.4^0. When the temperature has risen to 40^0C, first order diffraction is observed at 30^0. Calculate the mean co-efficient of linear expansivity of the crystal for the temperature range 0^0C to 10^0C.

Solution

At 0^0c, n = 1, θ = 30.4^0, 2dsinθ = nλ $2d\sin\theta=\lambda$ $d_0=\frac{\lambda}{2\sin 30.4}=\frac{\lambda}{1.01}$......1

At 40^0C, n = 1, θ = 30, $d_{40}=\frac{\lambda}{2\sin 30}=\lambda$..........2, but using $d_\theta=d_0(1+\alpha\theta)$,

$$\alpha=\frac{d_\theta-d_0}{d_0\theta}=\frac{\lambda-\frac{\lambda}{1.01}}{\frac{\lambda}{1.01}x40}=$$

7. Calculate the smallest glancing angle at which the x- rays of wave length $7x10^{-11}$m will be reflected from quartz which has an atomic separation of $1.5x10^{-10}$m. What is the highest reflection order that can be observed with this deflection?

Solution

The smallest glancing angle occurs when $\sin\theta=-1$. Highest order occurs when $\sin\theta=1$, hence $n_{\max}=\frac{2d\sin\theta}{\lambda}=\frac{2x1.5x10^{-10}}{7x10^{-11}}=4$

8. A beam of x- rays of wave length 0.20nm is incident on a crystal at glancing angle of 30^0. If the inter planar separation is 0.20nm, find the order of diffraction.

9. Show that the density ρ of a crystal can be given by $\rho=\frac{M\sin^3\theta}{125N_A(n\lambda)^3}$, where θ is the glancing angle, n is the order of diffraction, λ is the X – ray wave length and M is the molecular weight of the crystal.

10. A monochromatic beam of x- rays of wave length $2.0x10^{-10}$m is incident on a set of cubic planes of a potassium chloride crystal. If the inter planar spacing is $3.1x10^{-10}$m and the molecular weight of potassium chloride is 74.5, find:
    i. The glancing angle for the first order diffraction maxima to be observed.
    ii. The density of potassium chloride.

11. (a) (i) Draw a labeled diagram of an x- ray tube

    (ii) Use the diagram in a (i) to describe how x – rays are produced.

    (iii) State one industrial and one biological use of x – rays

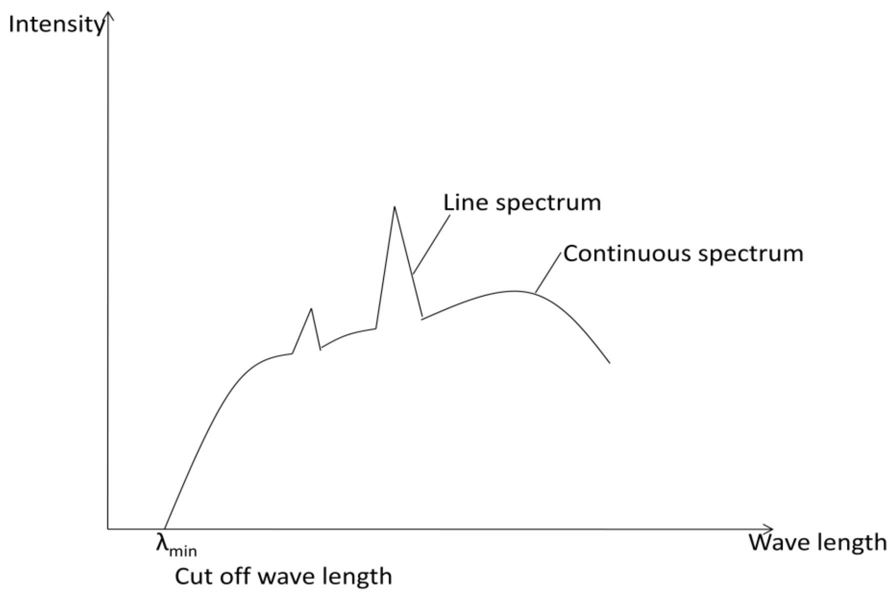
    (b) Sketch a graph of intensity versus wave length of x – rays from an x-ray tube and describe it.

X-ray emission spectra

The kinetic energy of an electron arriving at the target is given by $\frac{1}{2}mv^2 = eV = hf$. This energy is lost due to rapid deceleration of the first moving electron on striking the metal target and released as electromagnetic waves of varying frequency. This radiation of continuously varying frequency is called the back ground of continuous radiation or white spectrum.

The x- ray spectra consists of two distinct components:

i. A back ground of continuous radiation, the minimum wave length of which depends on the operating voltage of the tube (energy of bombarding electrons)
ii. Very intense emission at few discrete wave lengths. These wave lengths are characteristic of the target material and are independent of the operating voltage.



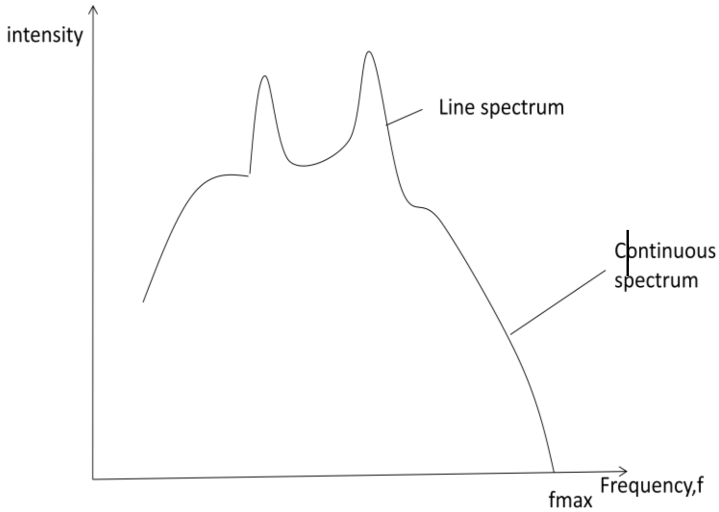
Main features of an x- ray spectra

a. Continuous spectrum
b. Definite cut off wave length.
c. Line spectrum.

N:B:

(i) For two accelerating potentials, the higher p.d is super posed to the lower p.d.
(ii) A graph of intensity against frequency can be plotted and is the opposite of the one for wave length.

Also, a graph of intensity against frequency appears as shown below



Explanation for the graph

The intensity of the continuous spsctrum having a definiten wave lengthn increases to maximum and decreases as the wave length becomes longer. Line spectrum of two or three peaks of high intensity are super imposed on the continuos spectrum. The peaks are of the series K, L and Mdepending on the target e;lement.

Continuous spectrum

Is as a result of the energized electron from the cathode hitting the target repeatedly before it loses all its kinetic energy as it is decelerated.The energy of the emitted x- ray is equal to the energy lost due to deceleration. If the intensity of the emitted x- ray radiation is plotted against frequency, thus a continuous curve will be obtained.

Definite cut off wave length

This is due to the electron losing all its kinetic energy in a single encounter to the atoms of the target. The x- ray photon produced has maximum energy and thus maximum frequency. There fore the wave length is minimum called cut off wave length.

Alternatively

If a direct collision is made with an atom and all the energy is absorbed, then on quantum theory, the x- ray quantum produced has maximum energy. But $hf_{max} = eV \Rightarrow f_{max} = \frac{eV}{h}$,

thus $\lambda_{min} = \frac{c}{f_{max}} = \frac{ch}{eV}$.

Line spectrum (emission spectra)

Is as a result of electron transitions within the atoms of the target material. The energetic electron may eject an electron from the inner most shell and raise the atom to an excited state since its energy is increased and becomes unstable. An electron from a higher shell may move into the vacancy created in its atom and this causes a high energy quantum of electromagnetic radiation. Since the energy levels are characteristic of the target atoms, so too are the x- rays produced.

How line spectra accounts for the existence of discrete energy levels

Atoms of a particular element always give radiation (line spectrum) of definite frequencies (wave length). In an excited atom, an electron may fall to a lower energy level through definite stages. Each electron transition involves the emission of electromagnetic waves whose frequency depends on the difference in energy.

THERMIONIC EMISSION

Definition of terms:

i. Thermionic emission is the process by which electrons are emitted from the metal surface which has been heated.
i. *Space charge* is a cloud of electrons (negative charges) around the cathode at low anode potential.
ii. *Saturation current* is the current that flows when all the electrons emitted by the cathode are attracted by the anode. This current remains constant eventually as the tube voltage increases.
iii. *Space charge limited current* is the current that flows through the diode at low voltages when the number of electrons emitted by the cathode per second is greater than that attracted to the anode per second. The current increases with increases in tube voltage.
iv. *Saturation anode potential* is the minimum potential at which the saturation anode current is obtained.
v. *Rectification* is the process of converting an alternating current to a direct current by use of diodes.

Mechanism

Metals contain free electrons in the lattice. When the temperature of the metal is increased, the thermal velocities and kinetic energy of the electrons will be increased.

The chance of electrons escaping from the attraction of the positive ions, fixed in the lattice, will then also be raised. This phenomenon is called thermionic emission.

The thermionic diode

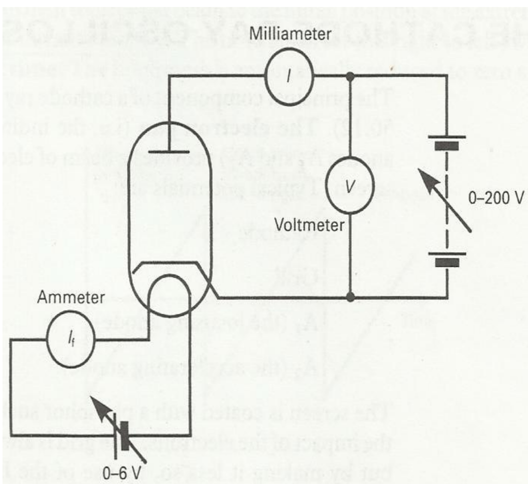
It consists of two electrodes ie anode and cathode. The cathode is heated and emits electrons by thermionic emission. When the anode is positive relative to the cathode, the electrons emitted by the cathode are attracted towards the anode and current flows.

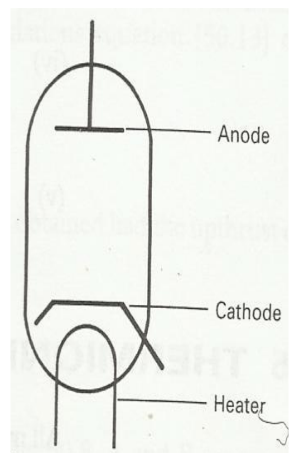
When the anode is negative relative to the cathode, the electrons are un able to reach the anode and there is no current flow.

N.B: it is normally referred to as a diode valve because it allows current to pass in one direction only.

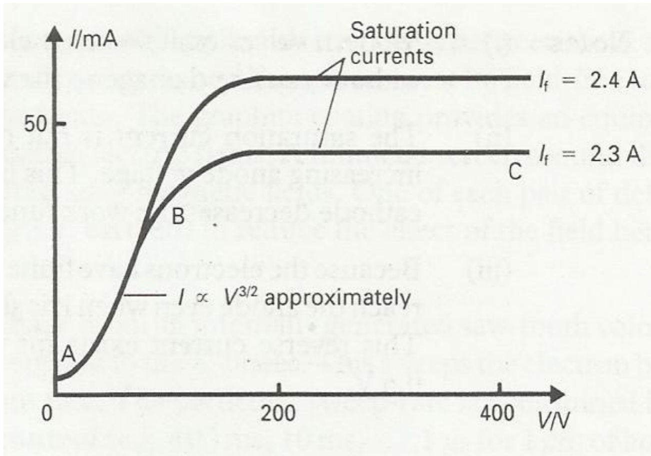
It is *used* to rectify alternating currents.

Circuit for investing diode characteristics **Circuit symbol**





Characteristics of a thermionic diode at two different temperatures



Explanation

2.4A represents higher temperature T_1 while 2.3A represents lower temperature T_2. ($T_1 > T_2$)

The current which flows through the diode is called anode current and depends on the p.d between the anode and the cathode. A graph of current I against voltage, V, appear as shown above for two different temperatures.

Explanation of the graphs

O to A: at V = o, electrons are emitted by the cathode with a range of velocities and some of the electrons have sufficient energy to reach the anode. Those emitted with low kinetic energy are prevented from reaching the anode because the electrons repel each other in the region between the cathode and the anode thus forming a space charge. This causes a small current to be detected.

AB: As Voltage increases, the electron density of the space charge reduces and its effect lowers. This increases the number of electrons reaching the anode thus increasing the current.

BC: As the voltage increases further, the space charge stops to exist and all the electrons leaving the cathode reach the anode. Any further increase in voltage has little effect on current and the current is said to have reached its saturation value.

NB:

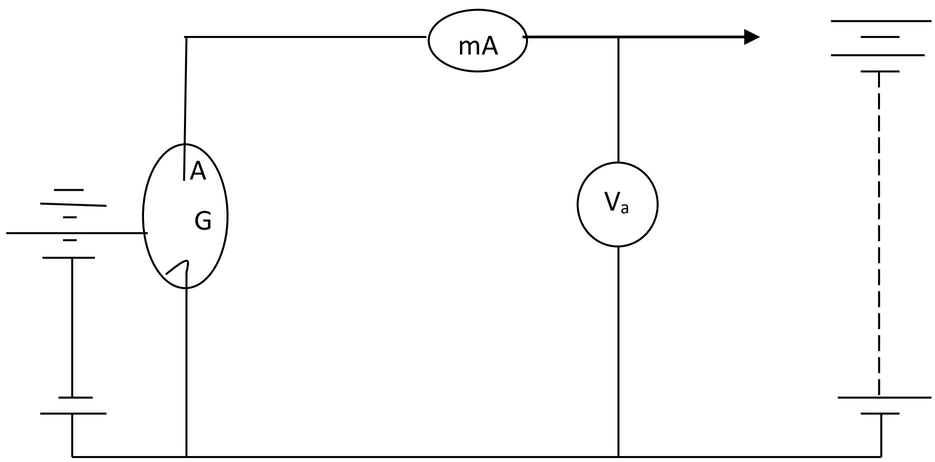
a. The current is then said to be *space charge limited* along AB and *temperature limited* along BC.

Triode valve

It consists of three electrodes that is cathode C, the emitter of electrons, anode A, the collector of the electrons and the gird G, a wire with open spaces placed between the cathode and the anode. The function of the gird is to control the electron flow to the anode, and for this purpose, the grid has a small negative potential relative to the cathode. The grid is nearer the cathode than the anode and its potential thus affects the electric field round the cathode more, with the result that the grid potential has a more delicate control than the anode potential over the anode current.

This enables the anode to act as an amplifier of alternating voltages as well as a detector.

Structure

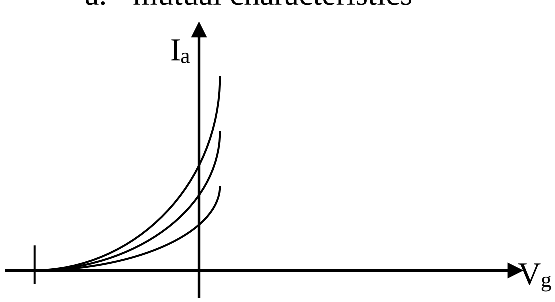


Triode valve characteristics

In order to predict the performance of a valve in a circuit, the characteristics of the valve must be first determined. The main characteristics are $I_a - V_g$ (V_a constant), variation of anode current with grid voltage when the anode voltage is constant. And $I_a - V_a$ (V_g constant), variation of anode current with anode voltage when grid voltage is constant.

The $I_a - V_g$ curves are known as the valve's mutual characteristics and $I_a - V_a$ curves are known as the anode characteristics.

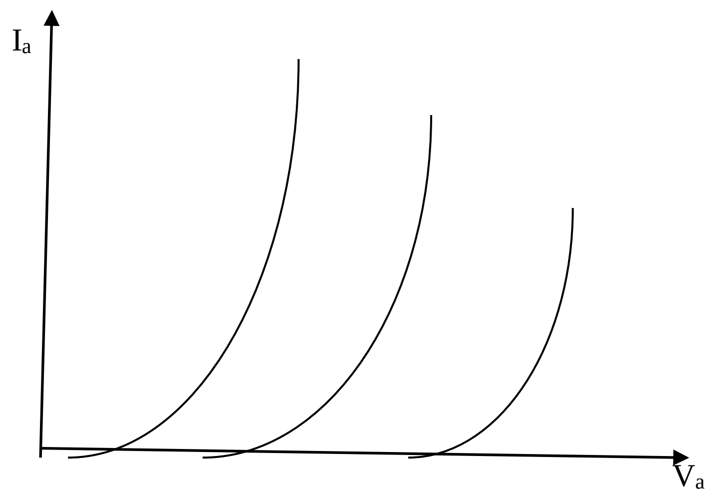
a. mutual characteristics



Explanation

When the high say 80V, a negative voltage on the grid such as -15V creates a resultant negative electric intensity at the cathode, and hence no electrons flow past the earth.
As the negative voltage is reduced and reaches a certain value, the attractive effect of the grid voltage and the electrons now reach the anode.
As the negative voltage is reduced further, more electrons reach the anode and the current increases as shown above.
The general shape of the $I_a - V_g$ curves is an initial curvature followed by a straight line.

b. Anode characteristics



Explanation

As the anode voltage V_a, increases, the anode current increases. Generally, the anode current begins to flow at higher values of $V_a$when the grid voltage is increased negatively.
If the anode voltage is increased sufficiently, all the electrons emitted by the cathode are collected and the current is said to have reached its saturation value.

Valve constants

There are three main constants or properties of a radio valve include:

i. Anode or A.C resistance, R_a, defined by $R_a = \frac{V_a}{I_a}(V_g, cons\tan t)$. The changes in Va and Ia being taken on the straight part of the anode characteristics.

ii. Mutual conductance, g_m, defined by $g_m = \frac{I_a}{V_g}(V_a, cons\tan t)$. The changes in V_g and I_a being taken on the straight part of mutual characteristics.

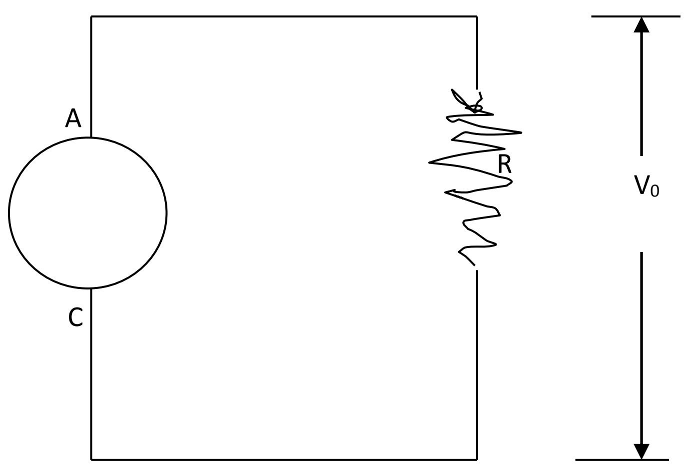
iii. Amplification factor or voltage gain defined by $\mu = \frac{V_a}{V_g}$

N.B:

a) R_a is the resistance of the valve when the anode circuit variations are considered.
b) g_m is the change in anode current produced by unit grid voltage variation.
c) μ is the measure of the step – up in voltage produced in the anode circuit by a change in grid voltage.

Triode as an amplifier

When the valve is used as a voltage amplifier in radio circuits, it is important to realize at the out set that it amplifies alternating voltages, and that these voltages are applied in the grid cathode circuit as shown below.



Voltage gain or amplification factor

The total resistance in the circuit is $R + R_a$. Thus the alternating current flowing $I = \frac{\mu V}{R + R_a}$.

Therefore, out put alternating voltage $V_0 = IR = \frac{\mu VR}{R + R_a}$

Voltage gain or amplification factor $= \frac{V_0}{V} = \frac{\mu R}{R + R_a}$.

Thus, if a triode has an amplification factor of 10, an internal resistance R_a of 8000Ω and a resistance R of 10,000Ω. Voltage gain $= \frac{\mu R}{R + R_a} = \frac{10x10000}{10000 + 8000} = 5.6$, hence if the applied alternating voltage is 0.2Vr.m.s, amplified out put voltage $= 5.6x0.2 = 1.1Vr.m.s$

Example

1. A triode valve with an anode resistance of 3000Ω is used as an amplifier. A sinusoidal alternating signal of amplitude 0.5V is applied to the grid of the valve. Find the r.m.s value of the output voltage if the amplification factor is 15 and anode load is 50KΩ.

Solution

$$\text{Voltage gain} = \frac{\mu R}{R + R_a} = \frac{15x50{,}000}{50{,}000 + 3000} = 14.15,$$

$$\text{Output voltage} = 14.15x0.5 = 7.075V_{r.m.s}$$

Assignment

2. Derive an expression for the amplification factor μ in terms of anode resistance R_a, and mutual conductance g_m for a triode valve. (answer: $\mu = R_a g_m$)
3. Derive an expression for voltage gain in terms of load resistance, R, anode resistance R_a, and mutual conductance, g_m. (answer: $\frac{R_a R g_m}{R_a + R}$)
4. A triode with mutual conductance of $3.0x10^{-3}AV^{-1}$ and anode resistance of 1000Ω, is used as a single stage amplifier. If the load resistance is 3000Ω, calculate the voltage gain of the amplifier. (answer: 2.25)
5. A triode with mutual conductance of $4.0mAV^{-1}$ and an anode resistance of 4KΩ is connected to a load resistance of 8KΩ. Assuming that the triode is operating under optimum conditions, estimate the output signal obtained for an alternating input signal of 20mV. (answer: $80x10^{-3}$mV)

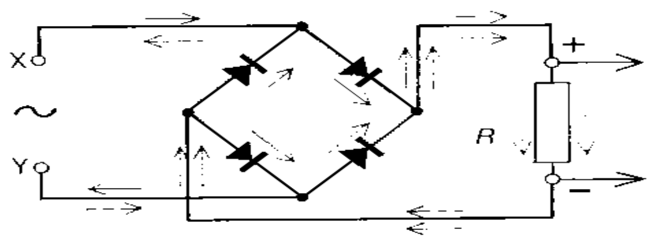
PROCESS OF RECTIFICATION .

With no diode , the voltage output across the load resistor , alternating current ,in put voltage = p.d across the resistor

Diag

With one diode, the out voltage is half way rectified on screen.

With four diode output voltage a full wave is rectified.



EXPLANATION

During half cycle, when A is positive and B is negative D4and D3 will conduct.

During the next half cycle if A is negative and B is positive D1and D2 conducts

In both half cycles current flows through R in one direction of a to b.

NUCLEAR STABILITY, FISSION AND FUSION

Einstein's mass energy relation

From Einstein's theory of relativity, mass and energy can be changed from one form to another. The energy E produced by a change of mass m is given by the relation $E = mc^2$, where c is the speed/ velocity of light numerically equal to $3x10^8 ms^{-1}$. If the mass m is in kg, then the energy E is in joules.

In electronics and nuclear energy, the unit of energy called an electro volt is normally used.itr is defined as the energy gained by an electron in being accelerated by a p.d of 1volt. $1eV = 1.6x10^{-19} J$. The mega electron volt (MeV) is a larger energy unit defined as 1 million electron volt. $1Mev = 1.6x10^{-13} J$

Unified atomic mass

If another unit of energy is needed, then one may decide to use a unit mass since mass and energy are interchangeable.

The unified atomic mass unit (u) is a twelfth $(\frac{1}{12})^{th}$ of the mass of one atom of carbon-12- isotope.

$12g$ of carbon contain $6.02x10^{12}$ atoms (Avogadro's number)

This implies that $6.02x10^{23}$ atoms weigh $12x10^{-3} kg$

1 atom will weigh $\frac{12x10^{-3}}{6.02x10^{23}} = 1.99x10^{-26} kg$

1u $= \frac{1}{12}$ of mass of 1 atom of carbon 12 isotope $= \frac{1}{12} x1.99x10^{-26} = 1.66x10^{-27} kg$

$\Rightarrow 1u = 1.66x10^{-27} kg$

But from the Einstein's theory of relativity, the mass energy relation is given by $E = mc^2$.

Since $1u = 1.66x10^{-27} kg$, $E = mc^2 = 1.66x10^{-27} x(3.0x10)^2 = 1.49x10^{-10} J$, but $1eV = 1.6x10^{-19} J \Rightarrow 1J =$

Thus $E = 1.49x10^{-10} J = 1.49x10^{-10} x \frac{1}{1.6x10^{-19}} = 9.31x10^8 eV = 931x10^6 eV = 931MeV$

Therefore $1u = 931MeV$

Binding energy

The protons and neutrons in the nucleus of an atom are called nucleons. The energy required to take all nucleons apart so that they are completely separated is called the *binding energy* of the nucleus.

Binding energy is the energy released when a nucleus is formed from its components (protons and neutrons)

The mass of the nucleus is always less than the total mass of its constituent nucleons. The difference in mass is called *mass defect.*

Mass defect is the difference in mass between the initial mass of the atom and final mass of the products. Therefore ***mass defect = mass of nucleons – mass of nucleus.***

The reduction in mass arises because the act of combining the nucleons to form the nucleus causes some of their mass to be released as energy.

Binding energy (J) = mass defect (kg) x c^2 (ms^{-1}). It follows therefore that

Binding energy (MeV) = 931 x mass defect (u)

Binding energy per nucleon is the ratio of the energy required to break the nucleus into its constituent nucleons to total number of nucleons (nuclear number).

Binding energy per nucleon = $\frac{binding energy}{total nucleons}$

Nuclear number is the number of nucleons (protons and neutrons) in the nucleus of an atom.

N.B: 1proton = $^{1}_{1}H$, 1 neutron = $^{1}_{0}n$, 1 electron = $^{0}_{-1}e$

A **radioactive** reaction procedes because the original nucleus has a greater mass than the mass of its products. Hence there is a mass loss, which manifests itself as energy.

This energy takes the form of the kinetic energy of the products of the reaction or if gamma rays are emitted, their photon energy(E = h*v*).

Similarly when a nucleus splits into two almost equal parts in a **fission** reaction. The mass of the original nucleus is greater than the sum of its products.

Fusion reactions obey the same rule. The masses of the original nuclei are greater than the mass of the nucleus they create.

Example

1. Calculate the binding energy and binding energy per nucleon of $^{59}_{28}Fe$ given the following information.

   Mass of $^{59}_{28}Fe$ nucleus = 58.93488u
   Mass of a proton = 1.00728u
   Mass of a neutron = 1.00867u
   1u = $1.66x10^{-27}kg$
   Solution

1proton = $^{1}_{1}H$, 1 neutron = $^{1}_{0}n$

Consider the equation below,

$$28\,(^{1}_{1}H) + 31\,(^{1}_{0}n) \rightarrow\ ^{59}_{28}Fe + \Delta m$$

Mass defect (Δm) = mass of nucleons – mass of nucleus

$$= 28x1.00728u + 31x1.00867u - 58.93488u$$
$$= 0.53773u = 0.53773x1.66x10^{-27}kg$$
$$= 8.92318x10^{-28}kg$$

Binding energy = $0.53773x931MeV = 500.62663MeV$

Binding energy per nucleon = $\frac{500.62663}{59} = 8.485MeV$

2. Calculate the binding energy for $^{59}_{26}Fe$, given the following information

   Mass of $^{59}_{26}Fe$ nucleus = 58.93488u

Mass of a proton = 1.00728u
Mass of a neutron = 1.00867u
1u = $1.66x10^{-27}\,kg$
Solution

1proton = $^{1}_{1}H$, 1 neutron = $^{1}_{0}n$

Consider the equation below,

26 ($^{1}_{1}H$) + 33 ($^{1}_{0}n$) → $^{59}_{26}Fe + \Delta m$

Mass defect (Δm) = mass of nucleons – mass of nucleus

$$= 26x1.00728u + 33x1.00867u - 58.93488u$$
$$= 0.54051u = 0.54051x1.66x10^{-27}\,kg$$
$$= 8.972\times10^{-27}\,kg$$

Binding energy = $0.54051x931MeV = 503.21MeV$

Binding energy per nucleon =$\frac{503.21}{59} = 8.53MeV$

Assignment

1. Can $^{210}_{84}Po$ decay to give off an alpha particle if $^{210}_{84}Po$ = 210.049u, $^{4}_{2}He$ = 4.0026u and $^{206}_{82}Pb$ = 206.034u? if possible, calculate:
   i. Mass defect.
   ii. Binding energy and binding energy per nucleon.
2. A uranium nucleus 238U emits an alpha particle according to the equation $238U \rightarrow ^{4}{}_{2}He + ^{234}{}_{92}Th + energy$.find the total energy released in joules. Take mass of 238U = 238.125u, mass of 234^{Th} = 234.117u, mass of 4He = 4.004u, 1u = $1.66x10^{-27}$kg.

   b) The isotope $^{238}_{92}\mathrm{U}$ emits an alpha particle and forms an isotope of thorium (Th), while the isotope $^{235}_{92}\mathrm{U}$ when bombarded by a neutron, forms $^{144}_{56}\mathrm{Ba}$, $^{90}_{36}\mathrm{Kr}$ and neutrons.Write the nuclear equations for the reactions of $^{238}_{92}\mathrm{U}$ and $^{235}_{92}\mathrm{U}$

3. 4pg of $^{235}_{92}U$ takes part in an atomic blast.

   Mass of proton = 1.00147u

   Mass of neutron = 1.00552u

Mass of $^{235}_{92}U$ = 234.9250u

1u = 1.66 x 10^{-27} kg

How much energy will be liberated?

4. Calculate the energy when a helium nuclei $^{4}_{2}He$ is produced by fusion of two deutronium nuclei. ($^{4}_{2}He$ = 4.004u, $^{2}_{1}H = 2.015u$, 1u = 1.66x10^{-27}kg)

5. Calculate the binding energy per nucleon of $^{7}_{3}Li$ isotope given

Mass of $^{7}_{3}Li$ = 7.07210u

Mass of proton = 1.0080u

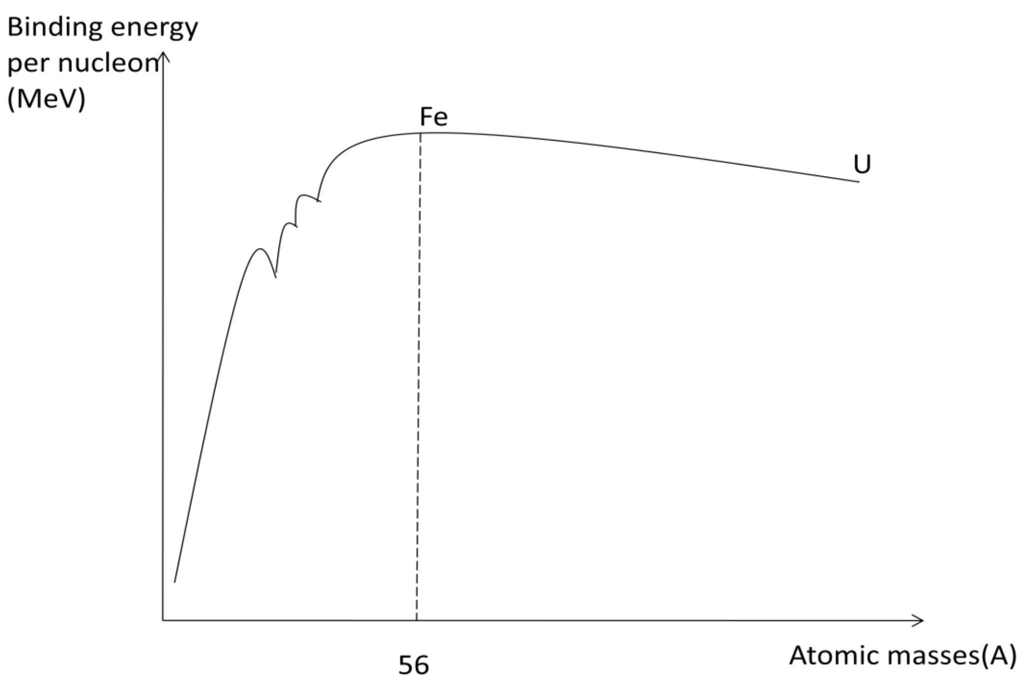
Mass of neutron = 1.0087u

Mass of electron = 0.0005u

1u = 932MeV

6. Calculate the energy in joules liberated when a helium nuclei $^{4}_{2}He$ is produced by fusion of two deutronium nuclei. ($^{4}_{2}He$ = 4.004u, $^{2}_{1}H$ =2.015u, 1u = 1.66x10^{-27}kg)
7. Calculate the energy released by fusing four protons to form an alpha particle and two beta particles.

Variation of binding energy per nucleon with mass number



Explanation

As the number of particles in the nucleus increases, the total binding energy per nucleon also increases. The binding energy per nucleon is a measure of stability of a nucleus and it represents the average energy required to remove a nucleon.

Binding energy of very small and very large nuclides is low and there are peaks for small nuclides for which the number of nucleons is equal to the number of protons.

The small values of binding energy per nucleon for higher and lower mass numbers imply that those nuclides are relatively unstable. At A = 56, the number of neutrons is large compared to the number of protons hence iron – 56 nuclide has the largest value of binding energy per nucleon.

Using the graph to explain how energy is released during fission and fusion

During fission, a heavy nucleus splits to form two lighter nuclei of greater binding energy per nucleon to make it more stable. The total mass of the two nuclei is less than the mass of the heavy nucleus. The difference in mass is accounted for by energy released.

During fusion, two lighter nuclei combine to form a heavier nucleus of greater binding energy per nucleon. The total mass of the bigger nucleus is less than the sum of the mass of the two nuclei. The mass difference is accounted for by the energy released.

Significance of binding energy per nucleon

(i) It's the measure of the stability of the nucleus.

(ii) The greater the binding energy per nucleon, the greater the work that must be done to remove the nucleon.

(iii) Maximum binding energy per nucleon corresponds to the most stable nuclei.

Energy of combination

When the nucleons come together in the nucleus, there is loss in energy equal to the binding energy. This results in a decrease in mass, from Einstein's mass energy relation. The decrease in mass is the difference in the mass of the individual nucleons when they are completely separated and the mass of the nucleus when they are together called mass defect.

Binding energy = mass of neutrons + mass of protons –mass of nucleus.

Question

1.(a) Define binding energy of nuclide.

(b) (i) Sketch a graph showing how binding energy per nucleon varies with mass number.

(ii) Describe the main features of the graph in b (i)

(c) Distinguish between nuclear fission and nuclear fusion and account for the energy released.

Energy of disintegration

Consider Polonium, $^{210}_{84}Po$. If an alpha particle could be emitted, $^{4}_{2}He$, and a lead nucleus $^{206}_{82}Pb$, a reaction could be represented by $^{210}_{84}Po \rightarrow {}^{206}_{82}Pb + {}^{4}_{2}He$.

Here the sum of mass numbers is 210 and the sum of nuclear charges is +84e of lead and helium respectively equal to the mass number and nuclear charge of the polonium nucleus, from the law of conservation of mass and energy.

If we require to find whether energy has been released or absorbed in the nuclear reaction, we should calculate the total mass of the lead and helium nuclei and compare this with the mass of the polonium nucleus. It is more convenient to use atomic masses rather than nuclear masses. These are as follows:

$^{4}_{2}He$ = 4.004u, $^{206}_{82}Pb$ = 205.969u, total mass = 209.973u, and $^{210}_{84}Po$ = 209.982u

Thus the atomic masses of the products of the reaction are together ***less than*** the original polonium nucleus, that is $^{210}_{84}Po \rightarrow {}^{206}_{82}Pb + {}^{4}_{2}He$ +Q, where Q is the energy released. Therefore polonium can disintegrate with emission of an alpha particle and release of energy that is polonium is un stable.

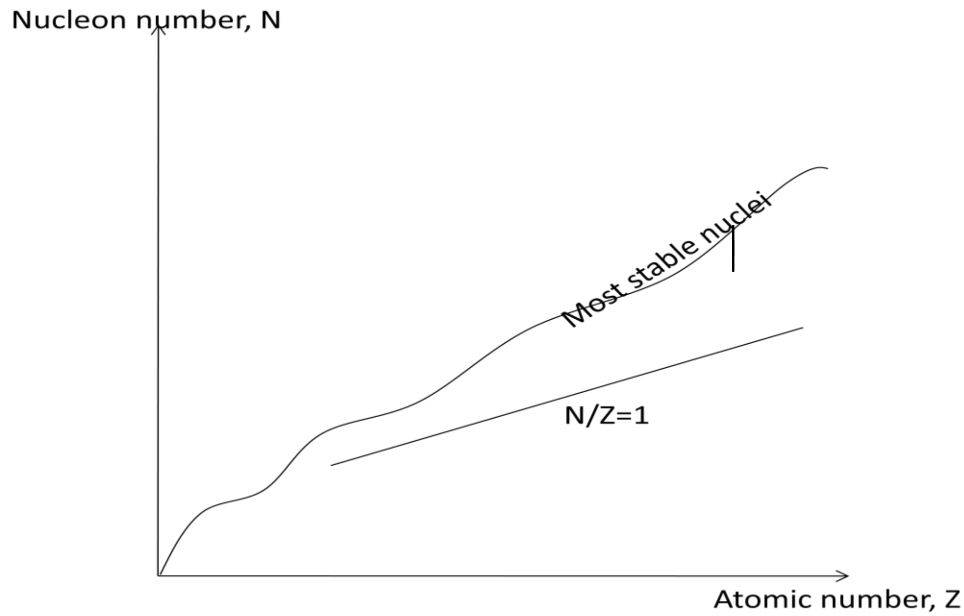
N.B: the reaction below is also possible,

$^{206}_{82}Pb$ + Q → $^{202}_{80}Hg + {}^{4}_{2}He$

Generally, a nucleus would tend to be *unstable* and emit an alpha particle if the sum of the atomic masses of the products are together ***less*** than that of the nucleus., and it would be *stable* if the sum of the atomic masses of the possible reaction products are together ***greater*** than the atomic mass of the nucleus.

Stable and unstable nuclei

There are many factors that contribute to the binding energy of the nuclei and there fore their stability.



Another factor that affects stability of nuclei is neutron to proton ratio. For light nuclei such as ${}^{12}_{6}C$ and ${}^{16}_{8}O$, this ratio is 1. For nuclei heavier than ${}^{40}_{20}Ca$the ratio $\frac{N}{Z}$ increases slowly towards about 1.6.There are stable nuclei above about Z = 92 (uranium).

Nuclear emission and nuclear stability

Unstable nuclei are radioactive. Their decay may occur in three main ways:

a. Alpha particle emission. If the nucleus has excess protons, an alpha emission would reduce the protons by 2 neutrons by 2. If a is the mass number and Z is the atomic number of the atom, ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$

b. β^- Particle (electron) emission. If the nucleus has too many neutrons for stability, the neutron – proton ratio is reduced by a beta particle emission. Here a neutron changes to a proton,
${}^{1}_{0}n \rightarrow {}^{1}_{1}H + {}^{0}_{-1}e$ (electron) hence A remains un changed but Z increases by 1.

c. β^+ Particle (positron) emission. If the nucleus is deficient of neutrons, decay by positron emission may occur. A proton changes to a neutron.${}^{1}_{1}H \rightarrow {}^{1}_{0}n + {}^{0}_{+1}e$(positron)
A is unchanged but Z decreases by 1.

Energy released in nuclear fission

Nuclear fission is the disintegration of a heavy nucleus into two lighter nuclei. Energy is released by the process of fission because the binding energy per nucleon of the fission products is greater than that of the parent. Energy is released when a heavy nucleus is split into two lighter nuclei.

Neutrons can used to produce nuclear disintegration. These particles are generally more efficient than protons for this purpose because *they have no charge and therefore able to penetrate more deeply into the positively charged nucleus*. This is the reason why neutrons are preferred to charged particles in nuclear reactions.

Also: because of the existence of the nuclear force of attraction which is even higher than that between electron and proton, then ***neutrons*** are also referred to as ***charged particles*** in nuclear reactions.

Example

1. Calculate the energy released when 1.05 kg of U-235 undergoes fission. Each fission produces 208 MeV.

$$N = 1\,750\ \cancel{g}\left(\frac{1\ \cancel{mol}}{235\ \cancel{g}}\right)\left(\frac{6.02\ x\,10^{23} nuclei}{1\ \cancel{mol}}\right) \quad = \quad 44.8\ x\,10^{23} nuclei$$

$$44.8\ x\,10^{23} \cancel{nuclei}\left(\frac{208\ \cancel{MeV}}{1\ \cancel{nuclei}}\right)\left(\frac{10^{6}\ \cancel{eV}}{1\ \cancel{MeV}}\right)\left(\frac{1.6\ x\,10^{-19}\ J}{1\ \cancel{eV}}\right)$$

$$= 14\,900\ x\,10^{10}\ J \quad = \quad \boxed{1.49\ x\,10^{14}\ J}$$

2. An unstable nucleus that is initially at rest decays into a nucleus of fermium-252 containing 100 protons and 152 neutrons and an alpha particle that has a kinetic energy of 8.42 *MeV*. The atomic masses of helium-4 and fermium-252 are 4.00260 *u* and 252.08249 *u*, respectively.

   a. What is the atomic number of the original unstable nucleus?

$$Z = 102$$

   b. What is the velocity of the alpha particle? (Neglect relativistic effects for this calculation.)

$$K = \frac{1}{2}mv^{2}$$

$$K = \left(8.42 \times 10^{6}\ \cancel{eV}\right)\left(\frac{1.6 \times 10^{-19}\ J}{1\ \cancel{eV}}\right) = 1.35 \times 10^{-12}\ J$$

$$m = (4\,u)\left(1.67\times10^{-27}\frac{kg}{u}\right) = 6.68\times10^{-27}\,kg$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(1.35\times10^{-12}\frac{kg\cdot m^2}{s^2}\right)}{\left(6.68\times10^{-27}\,kg\right)}} = 2.01\times10^{7}\,m/s$$

c. Where does the kinetic energy of the alpha particle come from? Explain briefly.

Mass Equivalence: The original nucleus decays into the product particles and energy. Energy Conservation: Potential or binding energy was converted into kinetic energy.

d. Suppose that the fermium-252 nucleus could undergo a decay in which a β^- particle was produced. How would this affect the atomic number of the nucleus? Explain briefly.

Atomic number increases by one. A neutron converts into a proton and an electron.

$$^{252}_{100}Fm \rightarrow \; ^{252}_{101}Md + \; ^{0}_{-1}e$$

3. A polonium nucleus of atomic number 84 and mass number 210 decays to a nucleus of lead by the emission of an alpha particle of mass 4.0026 atomic mass units and kinetic energy 5.5 *MeV*. (1 *u* = 931.5 MeV/c^2 = 1.66 x 10^{-27}*kg*.)

a. Determine each of the following.

i) The atomic number of the lead nucleus

Number of Protons $84 - 2 = 82$

i. The mass number of the lead nucleus

Number of Nucleons $210 - 4 = 206$

b. Determine the mass difference between the polonium nucleus and the lead nucleus, taking into account the kinetic energy of the alpha particle but ignoring the recoil energy of the lead nucleus.

The kinetic energy of the alpha particle is the mass difference of the two nuclei.

$$\Delta E = (\Delta m)c^2 \quad \Delta m = \frac{\Delta E}{c^2} \qquad \Delta m = \frac{5.5MeV}{c^2}$$

$$\Delta m = 5.5\frac{MeV}{c^2}\left(\frac{1.66\times10^{-27}kg}{931.5\frac{MeV}{c^2}}\right) = 9.8\times10^{-30}kg$$

c. Determine the speed of the alpha particle. A classical (non relativistic) approximation is adequate.

$$K = 5.5\,MeV\left(\frac{1\times10^6 eV}{1\,MeV}\right)\left(\frac{1.60\times10^{-19}J}{1\,eV}\right) = 8.8\times10^{-13}J$$

$$K = \frac{1}{2}mv^2 \quad v = \sqrt{\frac{2K}{m}}, \quad v = \sqrt{\frac{2(8.8\times10^{-13})}{6.64\times10^{-27}}} \qquad = 1.63\times10^7\,m/s$$

ii) Calculate the energy released when 3.8kg of ${}^{235}_{92}U$ under goes fission according to

${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n + \Delta E$

(Mass of ${}^{235}_{92}U$ = 235.04u, ${}^{141}_{56}Ba$ = 140.91u, ${}^{92}_{36}Kr$ = 91.91u, ${}^{1}_{0}n$ = 1.01u, 1u = 931MeV, Avogadro's number = $6.02x10^{23}$ atoms)

Solution

Mass defect = $235.04 + 1.01 - (140.91 + 91.91 + 3x1.01) = 0.20u$

Energy released $= 0.20x931 = 186.2MeV$

But $235x10^{-3}\,kg$ of U contain $6.02x10^{23}$ atoms

3.8kg will contain $\frac{3.8x6.02x10^{23}}{235.04x10^{-3}} = 9.73x10^{24}$

Energy released by 3.8kg of U will be = $\frac{9.73x10^{24}x186.2}{6.02x10^{23}} = 3009.5MeV$

Exercise

1. Explain why energy is released during nuclear fission and why it occurs with heavy nuclei.
2. Explain why the mass of the nucleus is always less than the mass of its constituent particles.

Energy release in fusion

Nuclear fusion is the combining of two lighter unstable nuclei to produce a heavier stable nucleus. A high temperature is required during fusion of nuclides because nuclei must first have very high kinetic energy to overcome electrostatic repulsion and this happens at high temperatures.

Example

1. During the fusion of Uranium, U – 235, 200Mev of energy is produced. Calculate the energy released when 2kg of Uranium takes part in bomb explosion

Solution

235u of uranium released 200Mev

1u of uranium release 931MeV

But 235g of uranium contain $6.02x10^{23}$ atoms

2kg = 2000g will contain $\frac{2000x6.02x10^{23}}{235} = 5.123x10^{23}$

Thus $6.02x10^{23}$ atoms released 200Mev

$5.123x10^{23}$ atoms will release $\frac{.5.123x10^{23}x200}{6.02x10^{23}} = 170.199MeV$

RADIOACTIVITY (RADIOACTIVE DECAY)

Is the spontaneous disintegration of a radioactive nuclide to emit alpha and beta particles and gamma rays. The elements that undergo radioactivity are called radioactive elements for example uranium, thorium, radium etc. These particles and radiations can be identified according to their properties. Radioactivity can be natural or artificial ie when a nucleus is bombarded with a neutron, proton, alpha particle of beta particle.

Radioactive decay is a *spontaneous* process because the rate of decay is proportional to the number of atoms present and the disintegration cannot be affected by temperature, pressure or any other factor (rate of decay is constant).

Atomic nucleus

This is made up three basic particles:

i. Positively charged protons
ii. Negatively charged electrons
iii. Neutral neutrons

The nucleus is made up of protons and neutrons while the electrons are located along the energy levels.

Atomic number (Z)

Is the number of protons in the nucleus of an atom.

Mass number (A)

Is the total number of protons and neutrons in the nucleus of an atom.

The neutrons are denoted by N. Thus A= z + N

An atom X is represented as ${}^{A}_{Z}X$

A nuclide is a specific nucleus of an atom characterized by its atomic number Z and mass number A.

Alpha (α) particles, ${}^{4}_{2}He$

These are positively charged helium nucleus.

Alpha emission: ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$

Properties

i. These are positively charged particles with a charge of 2e.
ii. They have low penetrating power.
iii. They are both deflected by both electric and magnetic fields.
iv. They produce intense ionization of the surrounding material through which they pass.
v. They are defected by electric and magnetic fields to a less extent than β particles.
vi. This means that they are heavier than β particles. In both magnetic and electric fields they are deflected in a direction opposite to that of the β particles. This indicates that they are positively charged.
vii. They are emitted with speeds of the order $10^{-7} ms^{-1}$
viii. They are helium nuclei with mass 4U .

ix. They have slight effect on the photo graphic plate.

Beta (β) particles ($_{-1}^{0}e$ or $_{-1}^{0}\beta$)

These are fast moving electrons.

Beta emission: ${}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e$

Properties

i. They are negatively charged.
ii. They have high penetrating power compared to alpha particles.
iii. They are deflected by both electric and magnetic fields more than alpha particles due to their small mass.
iv. They have very slight effect on the photo g\graphic plate.
v. They have low ionizing potential compared to alpha particles.

Gamma (γ) rays ($_{0}^{0}\gamma$)

These are electromagnetic radiations of very short wave length.

Gamma emission: ${}_{Z}^{A}X \rightarrow {}_{Z}^{A}X + {}_{0}^{0}\gamma$

Properties

They have no charge.

They are not affected by both electric and magnetic fields.

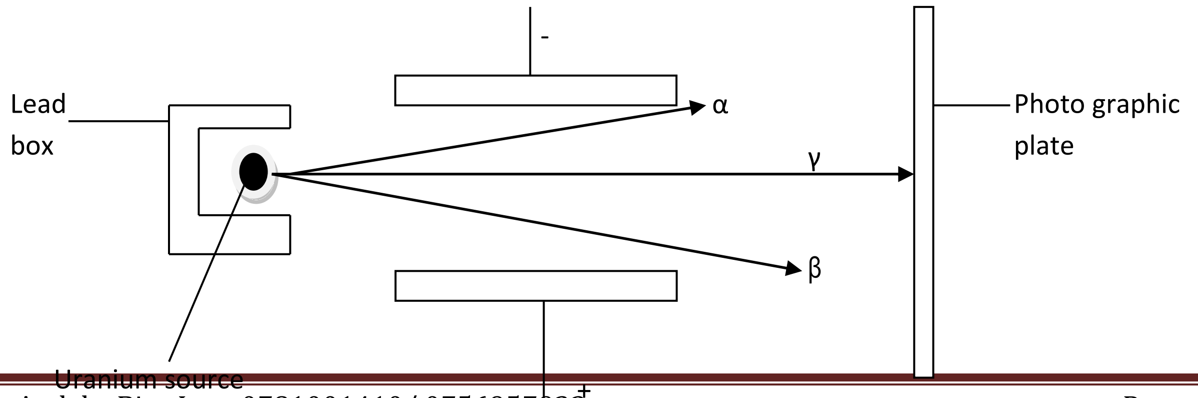
They have a high penetrating power than alpha and beta particles.

Have low ionization potential.

Completely destroy the photo graphic plate.

They travel at the speed of light.

Experiment to distinguish between alpha, beta particles and gamma rays



The properties of α and β particles and γ rays can be studied using a lead box containing a radioactive substance such as uranium. It is observed that β particles are deflected more than α particle because of the small mass and because of the high velocity of γ rays, they are not deflected.

Uses of radioactivity

SOME USES OF RADIOISOTOPES (RADIOACTIVITY)

1. They are used in carbon dating to determine the age of fossils. (Biological use).
2. Monitoring the thickness of paper (or plastic or metal sheet) by making the paper sandwiched between a beta source and the detector. The thickness is controlled by measuring the transmission of radiation through the sheet. (Industrial use).
3. Sterilization of medical instruments and bandages by exposing them briefly to gamma rays after packing. (Biological use). Gamma ray as are also being used to sterilize and preserve some food products. The method is safe as no radioactivity is induced in the material irradiated by γ- rays.
   Radiation is also been used to eliminate agricultural pests by sterilizing them.
4. Tracing leakage in water pipes by putting a small amount of sodium 24 into the water flowing. This causes the activity to build up in the soil around the leakage point and this can be detected without having to dig the whole pipe line out. (Industrial use).
5. Assessment of blood volume in a patient can be estimated by introducing a radioactive substance in the patient. Then the activity of the blood after a known time is found and the blood volume is computed. (Biological use).
6. Cancerous cells can be destroyed by γ –radiation from high activity source of cobalt 60. This is called radiotherapy. (Biological use).

Effects of radiations

i. They damage the skin tissues leading to skin cancer.
ii. They lead to heredity effects in genetics (mutation).
iii. Cause deep seated wounds.

Precautions

i. Avoid un necessary exposure to radiations.
ii. Avoid ingestion of any radioactive material.
iii. Avoid inflation of radiations.

Exercise

1. Give concisely the important facts about mass, charge and velocity associated with α, β and γ ray emission respectively.

Detectors of radiation

These include:

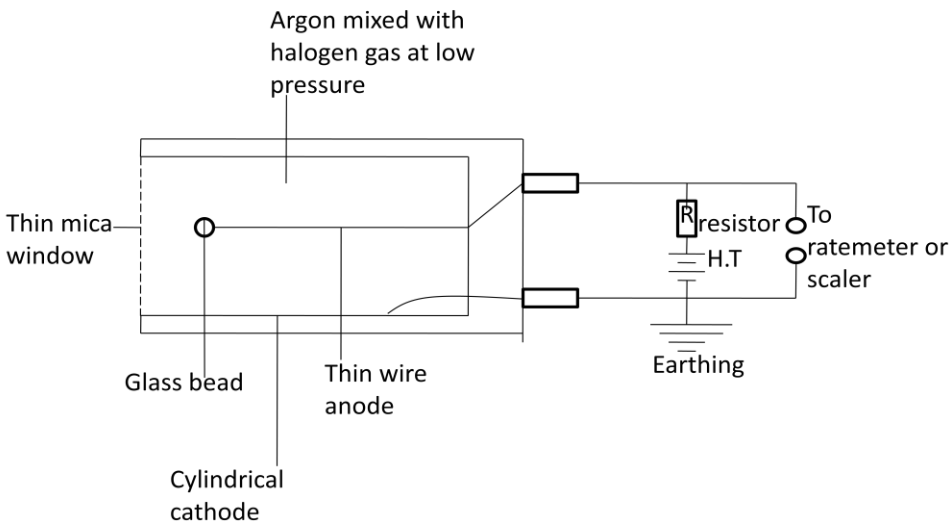
i. Geiger Muller tube (G.M.T)
ii. Diffusion cloud chamber
iii. Wilson's cloud chamber.
iv. Ionization chamber

i. Geiger Muller tube (G.M.T)

A Geiger Muller tube can be used to detect the presence of X- rays, γ rays and β particles.

Structure

It consists of a thin central wire anode insulated from the surrounding by a cylindrical cathode. The entrance of the tube is made of a thin cylindrical mica window.



Action

When an ionizing particle enters the tube, ion pairs are produced through the collisions with argon atoms.

The electrons produced are accelerated towards the anode and more ion pairs are produced by repeated collision, which electrons so produced produce more electrons resulting into an avalanche or gas amplification.

On reaching the anode, discharge occurs and causes a current pulse to flow through R.

The voltage pulse which develops is amplified and operates the counter.

The positive ions which would have caused second ionization only discharge on reaching the cathode and are slowed down by collision with bromine molecules.

The saturation current is proportional to the energy of incident ionizing particle.

N.B

i. The anode is made thin to increase the electric field/ strength around it since electric field intensity is proportional to the radius of the wire.

ii. The mica window is made thin to allow even the weakest ionizing agent to enter into the tube.

iii. Argon at low pressure enables formation of ion pairs when particles collide with neutral atoms.

iv. Halogen gas forms a quenching agent to prevent secondary ionization. It also ensures that only one current pulse is produced by only one particle that enters the G.M tube.

    How it works

    The halogen vapor absorbs the energy of the positive ions as a result of collision with the halogen molecule before they reach the cathode. Therefore their energy is used to dissociate the halogen molecules instead of releasing electrons from the cathode.

v. An avalanche is a cloud of negative ions around the anode.

vi. Dead time is the time over which the tube does not respond at all to the arrival of the ionizing particles.

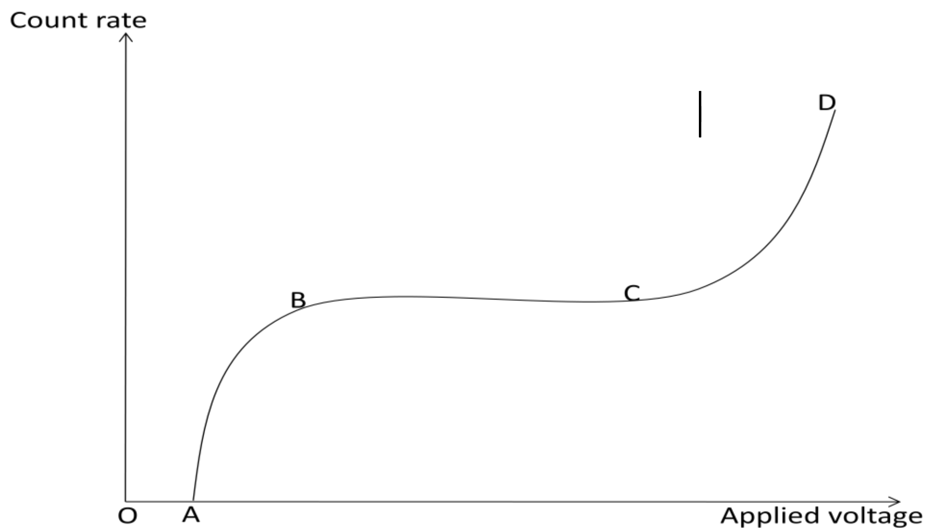
vii. Recovery time is the time over which the pulses are produced but not large enough to be detectable.

viii. Pressure of an inert gas is important in generation of avalanches.

ix. The glass bead prevents sparkling.

x. **Threshold voltage** is the voltage below which there is not sufficient gas amplification to produce pulse high enough to be detected

Variation of count rate with applied p.d in G.M.T (characteristics of G.M.T)



OA – the operating voltage here is not enough to attract the ions to the respective electrodes and hence the counter registers no reading. This voltage is called the threshold voltage.

AB – is the proportional region. Increasing the anode and cathode potentials beyond the threshold voltage increases the activity of the GMT and hence an increase in the count rate between O^I and A.

BC – here the count rate is constant. This is called the plateau region. Between BC, all the negative ions are able to reach the anode because the operating voltage is large enough to attract them. Here the tube is said to be operating normally.

Every particle which produces any ionization at all is detected. A full avalanche is obtained along the entire length of the anode

CD: in this region, the anode potential is so large that the electrons moving towards the anode collide with other ions or atoms because of their rapid motion. This collision of the ions causes avalanche of electrons thus causing secondary ionization. When secondary ionisations take place, a bigger ionization current flows. The current increases and the detector falsely registers a high count rate.

Secondary ionization is eliminated by a quenching agent which ensures that for every ionizing particle that enters the tube, one pulse is produced.

NOTE:

The suitable region for operating the tube is along AB because within this region every particle that produces ionization is detected.

Threshold voltage is the p.d below which no count rate is registered by the counter. It is the p.d below which there is no sufficient gas amplification to produce current pulses large enough to be detected because the counting device in the circuit is designed in such a way to register pulses above certain magnitude.

Question

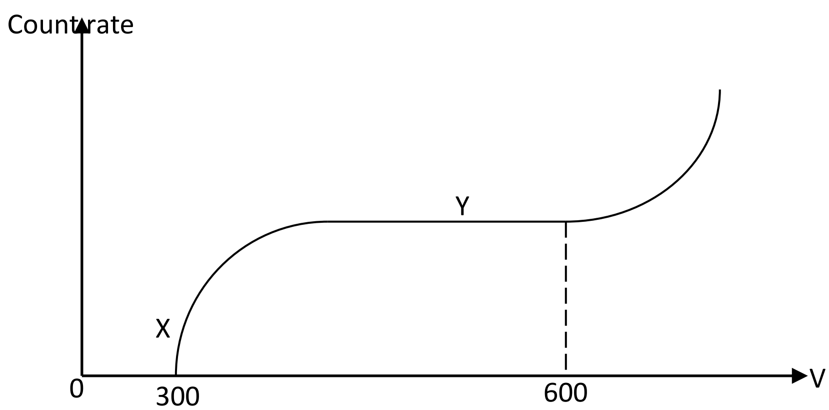
a) With the aid of a labeled diagram, describe the working of the Geiger – Muller tube.

b) How would you use a Geiger – Muller tube to determine the half life of a radioactive sample?

c) (i)Sketch the count rate – voltage characteristics of the Geiger Muller tube and explains it main features.

(ii)Identity giving reasons the suitable range in b (ii) above.

d) The figure below shows roughly how the count rate of a Geiger Muller tube varies with the voltage V applied to the anode of the tube.



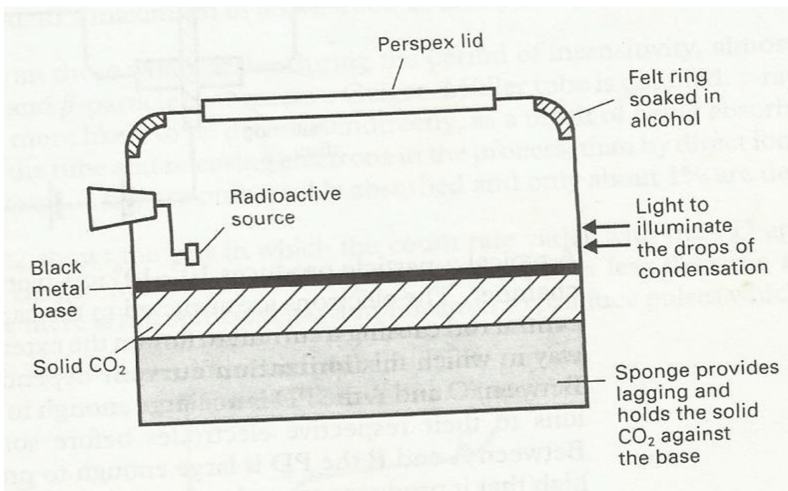
i) What is the most suitable voltage approximately for operating the GM tube?

ii) Explain the existence of the plateau Y in graph of plateau in graph.

ii The diffusion cloud chamber

The cloud chamber displays tracks of any ionizing agents which pass through it.

Structure



Action

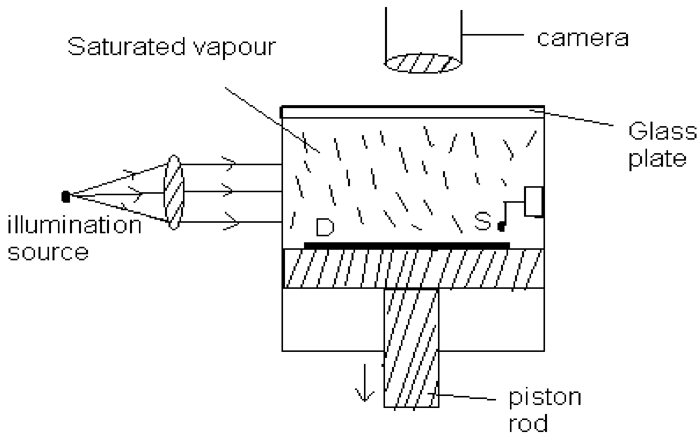
Alcohol in the upper most part is vaporized so as to diffuse into the cold system of the chamber.

This results into the formation of a super saturated alcohol layer just above the cold system leaving the air above free.

This free air is subjected to controlled ionization by radiations from the source.

When the ionized air is illuminated and photo graphed, alpha particles appear dense straight tracks, beta particles leave thinner tracks.

Wilson cloud chamber (expansion cloud chamber)



Action

When the piston is moved down quickly, the air in the chamber undergoes adiabatic expansion and cools.

The air then becomes super saturated.

The air is exposed to alpha particles from a radioactive source.

Water droplets collect round the ion pairs which act as centres of formation.

The drops are illuminated, and photographed by light scattered from them.

Alpha particles produce continuous straight paths, beta particle produce longer but straggly paths.

Example

A certain alpha particle in a cloud chamber has a length of 4cm. Given that the average energy required to produce an ion pair is $5x10^{-18}$J and that alpha particle in air produce an average of $6.0x10^{3}$ such ion pairs per cm track.

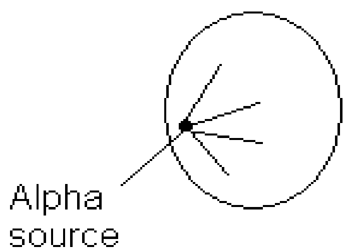
a) Find the initial energy in MeV.

b) Explain why the number of ion pairs is likely to increase towards the end of the track of an alpha particle.

NATURE OF TRACKS

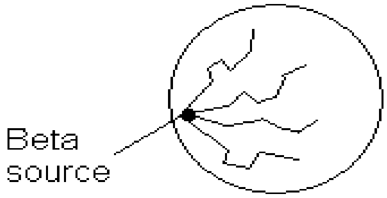
ALPHA PARTICLES

- cause intensive ionization which leads to thick tracks
- they are of short range (short track)
- they are emitted with the same kinetic energy from any given source (tracks of the same length)
- they have a high momentum and therefore are not deflected by collisions with vapour atoms (straight tracks)



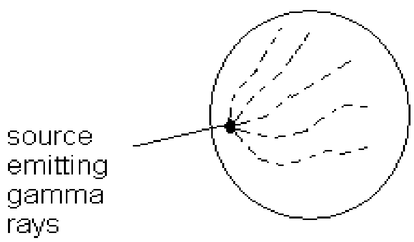
BETA PARTICLES

- more penetrative than alpha particles (long tracks)
- less ionizing than alpha particles (thin tracks)
- light particles moving at high speed (tracks are originally straight, there after become erratic)
- emitted with varying speeds from any given source(tracks of varying lengths)



GAMMA RAYS

-they have a very weak ionization potential and very penetrative



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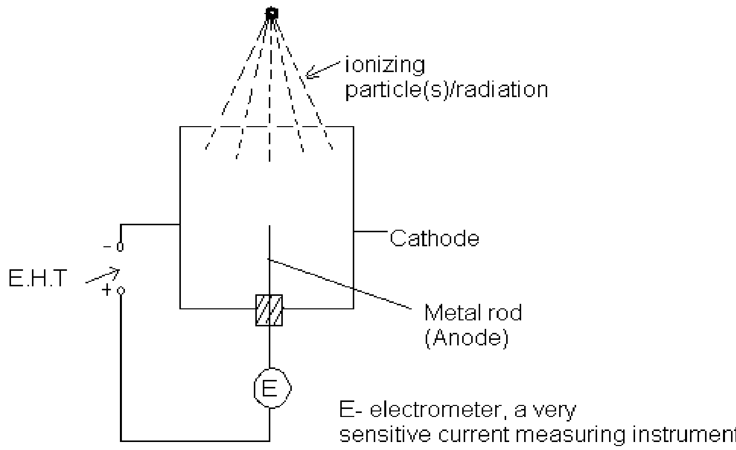
Effect of reducing pressure in the cloud chamber on length of tracks.

The length increases because the rate of collision of alpha particles with air is reduced therefore reduces the rate of momentum transfer.

iii **Ionization chamber**

Structure



Action

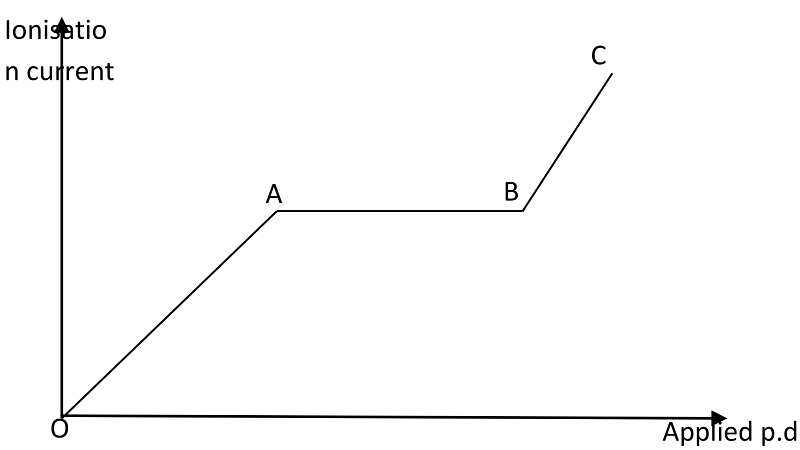
When the radiation enters the chamber, argon atoms at low pressure are ionized.

The positive ions move towards the cathode while the negative ions move towards the anode.

This discharge sets up a current in the external circuit such that a small voltage drop a cross external register R develops.

This voltage is amplified and passed to the counter which registers the passage of ionizing particle that enters the chamber.

Characteristics graph / curve of ionization current against applied p.d



Explanation

O to A

The p.d is not large enough to draw all the electrons and positive ions to their respective electrodes. As p.d increases, more ions reach the electrodes thus increasing the current.

A to B

All the ions are attracted to their respective electrodes and there is no recombination. The current remains constant as p.d increases.

Beyond B

Current increases rapidly for small change in potential difference and it is as a result of secondary ionization which takes place leading to many ion pairs. Thus large current being produced.

Example

The energy required to produce one ion pair in air is 32eV. Calculate the current that flows when one alpha particle per second from radioactive source is stopped in the ionization chamber if the energy of that alpha particle is 5.2MeV.

Solution

Let n be the number of ion pairs produced per second $2ne = 2 \times 10^{-7} \Longrightarrow n = \frac{2\times10^{-7}}{2\times1.6\times10^{-19}} 6.25x10^{9} pairs\ per\ second.$

$$energy\ lost\ per\ second = 6.25x10^{9}x32MeV = 2x10^{11}eV$$

Energy of one alpha particle $= \frac{2x10^{11}}{2x10^{5}} = 1.0MeV$

Isotopes

These are atoms of the same element having the same number of protons but different number of neutrons (mass number).for example Cl – 37 and Cl – 35.

Radioisotopes: are radioactive atoms of an element that have the same atomic number but different mass numbers.

Uses of radioisotopes

i. Biological use: they are used in radiotherapy, tracers, stylization and carbon dating.
ii. Industrial uses: used as tracers and diagnostics.

Other uses:

a. Used in carbon dating.
b. Used to determine the thickness of paper.

Radioisotopes are nuclides which are unstable and undergo radioactive decay emitting $\propto$ or β particles or γ- rays during return to a stable form. ^{238}U, ^{226}Ra and ^{230}Th are examples of natural radioisotopes.

A greater number of radio isotopes are produced artificially by bombarding stable nucleus with high energetic particles such as protons, α-particles, deuterons and neutrons. Artificial radioisotopes behave the same way as the natural radioactive materials in that each will emit its characteristic particle or radiation and each has a characteristic half-life.

Decay laws

1. ***The law of conservation of nucleons.***

It states that the total number of protons and neutrons entering a reaction must be equal to the total number of nucleons leaving the reaction.

$$Nucleons = Number of\ protons + number of\ protons$$

2. ***The Decay law***

States that the rate of disintegration of an atomic nucleus of an element is directly proportional to the number of atoms present ie $\frac{dN}{dt} \alpha N$ (separating variables and integrating both sides)

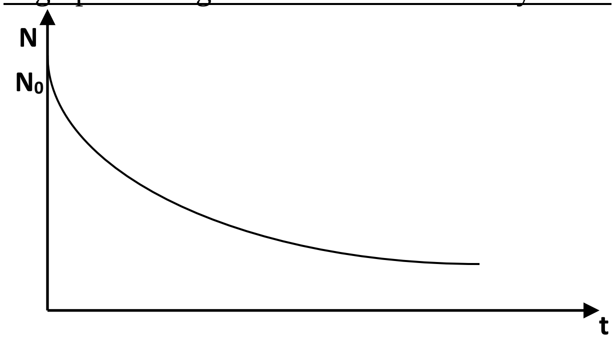
$$\frac{dN}{dt} = -\lambda N$$

$$\Rightarrow \int_{N_0}^{N} \frac{dN}{N} = \int_0^t -\lambda dt \Rightarrow In[N]_{N_0}{}^{N} = -\lambda t$$

$$InN - InN_0 = -\lambda t \Rightarrow In\left(\frac{N}{N_0}\right) = -\lambda t \Rightarrow \frac{N}{N_0} = e^{-\lambda t} \Rightarrow N = N_0 e^{-\lambda t}$$

Where λ is a constant characteristic of the atom concerned called the radioactivity decay constant. The negative sign indicates that N becomes smaller when t decreases.

A graph of N against t is called the decay curve



Activity

It is the number of disintegration per second. One disintegration per second is called a Becquerel (1Bq). The Becquerel is the S.I unit of activity.

The larger unit of activity is the curie (Ci)

1Ci = 3.7 x 10^{10} Bq = 3.7 x 10^{10} disintegrations per second

$$A = -\frac{dN}{dt}, \text{The negative is because each decay means a reduction of the value of } N.$$

$$But \frac{dN}{dt} = -\lambda N$$

$$-\frac{dN}{dt} = \lambda N$$

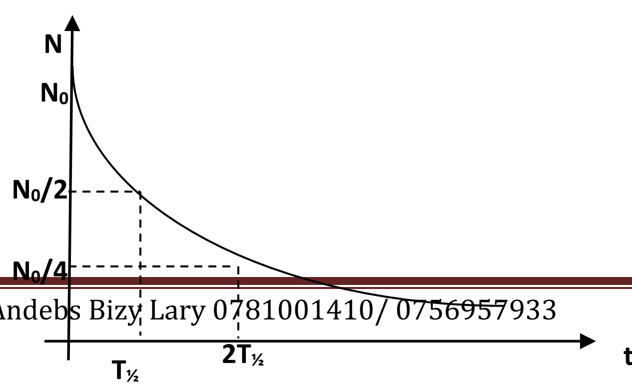
$$\therefore A = \lambda N \text{ and } \lambda = -\frac{1}{N}\left(\frac{dN}{dt}\right) \ (\text{the decay constant})$$

$$\text{let } A_0 \text{ be the activity at } t = 0, \text{then } A_0 = \lambda N_0$$

$$From \ \ N = N_0 e^{-\lambda t}$$

$$A = A_0 e^{-\lambda t}$$

Consider the decay curve of a radioactive source



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Definition: **decay constant** is the ratio of activity to the number of active atoms present or the fractional number of disintegrations per second.

- The probability that any one nucleus will decay within the time interval **dt** is thus:

$$\text{probability} = \frac{dN}{N} = \lambda dt$$

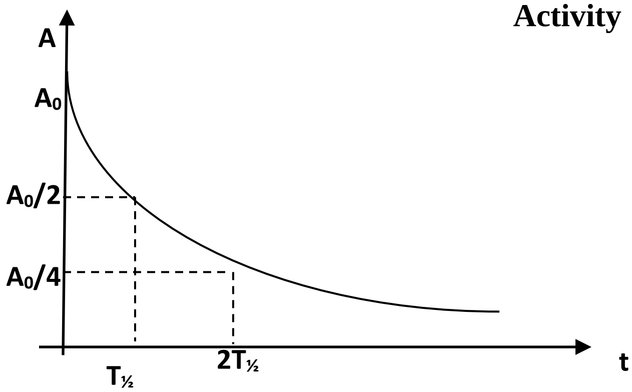
and so the probability of decay per unit time is equal to the **decay constant:**

$$\frac{\text{probability}}{dt} = \lambda$$

Also: $N_0\lambda = A_0$, thus $A = A_0 e^{-\lambda t}$

A graph of activity against time t has the form

Activity



Half life $\left(t_{\frac{1}{2}}\right)$

Is the time taken for half the atoms of a radioactive element to disintegrate or

Is the time taken for half the nuclei present to disintegrate.

From the decay law, $N = N_0 e^{-\lambda t}$

For half life, $N = \frac{N_0}{2}, t = t_{\frac{1}{2}}$ thus $\frac{N_0}{2} = N_0 e^{-\lambda t} \Rightarrow \frac{1}{2} = e^{-\lambda t_{\frac{1}{2}}}$ 1

Introducing natural logarithm (In) on both sides of equation 1 gives

$In\left(\frac{1}{2}\right)=-\lambda t_{\frac{1}{2}} \Rightarrow In1-In2=-\lambda t_{\frac{1}{2}}$, but In1 = 0, thus $In2=\lambda t_{\frac{1}{2}} \Rightarrow t_{\frac{1}{2}}=\frac{In2}{\lambda}$. Since In2 = 0.693, then the equation for half life can also be written as $t_{\frac{1}{2}}=\frac{0.693}{\lambda}$

Example:

1. The activity of charcoal taken from an ancient home stead is $4.8x10^{-19}Ckg^{-1}$. Given that the half life of carbon 14 found in living specimen is $7.0x10^{-19}Ckg^{-1}$. find the time when the home stead was used.

   Solution

   Using $A=A_0e^{-\lambda t}$, and $\lambda=\frac{0.693}{t_{\frac{1}{2}}}=\frac{0.693}{5760x365x24x3600}=3.815x10^{-12}s^{-1}$

   $4.8x10^{-19}=7x10^{-19}e^{-3.815x10^{-12}t} \Rightarrow t=9.889x10^{10}s$

2. At certain time, an alpha particle detector registers a count rate of 385 per second. Exactly 10 days later, the count rate dropped to 85 per second. Find the decay constant.

   Solution

   Using, $A=A_0e^{-\lambda t} \Rightarrow \lambda=\frac{1}{t}In\left(\frac{A_0}{A}\right)=\frac{1}{10}In\left(\frac{385}{85}\right)=0.151day^{-1}$

3. Lanthanum has a stable isotope 139_{La} and a radioactive isotope 138_{La} of half life $1.1x10^{10}$ years whose atoms are 0.1% of those of the stable isotope. Estimate the rate of decay of 138_{La} with 1kg of 139_{La} (Avogadro's number = $6.02x10^{23}mol^{-1}$).

   Solution:

   Decay rate $\frac{dN}{dt}$ = λN, where λ is the decay constant and N is the number of atoms in 138_{La}

   Number of atoms in 1kg of 139_{La} $\frac{6.02x10^{23}x1}{0.139}$

   Since 0.1% = 0.001, then number of atoms in 138_{La}, N = $\frac{0.001x6.02x10^{23}x1}{0.139}=\frac{6.02x10^{23}}{139}$

   Also, $\lambda=\frac{0.693}{t_{\frac{1}{2}}}=\frac{0.693}{1.1x10^{10}365x24x3600x139}$

   Rate of decay, $\frac{dN}{dt}=\frac{0.693x6.02x10^{23}}{1.1x10^{10}x365x24x3600x139}=8600s^{-1}$

4. When 238U (Uranium) decays, the end product is 206Pb (lead), the half life is $1.4x10^{17}$seconds. Suppose a rock sample contains 238U and 206Pb in the ratio of 5:1 by weight, calculate:

i. The number of Pb atoms in 1.0g of the rock sample.

ii. The age of the rock assuming the radioactive decay law is $N = N_0 e^{-\lambda t}$.

Solution

i. U:Pb = 5:1, mass of Pb $= \frac{1}{6} x1g = \frac{1}{6} g$, mass of U $= \frac{5}{6} x1g = \frac{5}{6} g$

1 mole of Pb contains $6.02x10^{23}$atoms, thus

206g of Pb contains $6.02x10^{23}$atoms

$\frac{1}{6} g$ Will contain $\frac{\left(\frac{1}{6}\right)}{206} x6.02x10^{23}$ atoms $= \frac{1}{6x206} x6.02x10^{23} = 4.87x10^{20}$ atoms.

ii. Using $t_{\frac{1}{2}} = \frac{0.693}{\lambda}$,

$\lambda = \frac{In2}{1.4x10^{17}} = 4.95x10^{-18} s^{-1}$, also $N = N_0 e^{-\lambda t}$,

Number of U atoms $= \frac{5}{238x6} x6.02x10^{23} = 2.108x10^{21} atoms$, this is the number of atoms for uranium left after a time t.

Therefore total initial number of atoms $N_0 = N_U + N_{Pb} = 2.108x10^{21} + 4.87x10^{20} = 2.595x10^{21} atoms$

Thus $t = -\frac{1}{\lambda} In\left(\frac{N}{N_0}\right) = -\frac{1}{4.95x10^{-18}} xIn\left(\frac{2.108x10^{21}}{2.595x10^{21}}\right) = 4.179x10^{16}$ seconds.

5. The activity of charcoal taken from a fire pit of an ancient homestead is $4.8 \times 10^{-9} Ci$ per Kg. Given that the half life of ^{14}C is 5760 years and that the activity of ^{14}C found in living specimens is $7.0 \times 10^{-9} Ci$ per Kg, find the time when the homestead was used.

SOLUTION:

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{5760} y^{-1}$$

$$In\left(\frac{A_0}{A}\right) = \lambda t \Rightarrow In\left(\frac{7.0}{4.8}\right) = \frac{0.693}{5760} t \Rightarrow t = \frac{5760}{0.693} In\left(\frac{7}{4.8}\right) years \quad t = 3136 years.$$

6. A small volume of a solution which contains a radioactive substance of sodium had an activity of 12000 disintegrations per second per minute when it was injected into the blood streams of a patient. After 30 years the activity of $1m^3$ of blood was found to be 0.5 disintegrations per minute. If the half life of the sodium isotope is 15 hours, estimate the volume of blood in the patient.

SOLUTION:

$$A = A_0 e^{-\lambda t} \Rightarrow A = 12000 e^{-\frac{In 2 \times 30 \times 60}{15 \times 60}} \Rightarrow A = 3000 \min^{-1}$$

Let V be the volume of blood in the patient

$$3000 \min^{-1} \rightarrow V cm^3$$
$$0.5 \min^{-1} \rightarrow 1 cm^3$$

$$\frac{3000}{0.5} = \frac{V}{1} \Rightarrow V = 6000 cm^3 .$$

Exercise

1. The activity of a dead sample of wood is 10 counts per minute while the activity of a living plant is 19counts per minute. If the half life of carbon-14 is 5500years. Find the age of the wood sample.(5092.997years)
2. The ratio of mass of lead- 206to the mass of uranium -238in the rock sample is found to be 0.55. Assuming that all the lead was originally uranium; calculate the age of the rock sample if the half life of uranium is $1.4 x 10^{17}$s.(9.25.10.16)
3. A sample from fresh wood of a certain species of tree has an activity of 16.0 counts per minute per gram. However, the activity of 5g of dead wood of the same species of tree is 10.0 counts per minute. Calculate the age of the dead wood. (assume half life of 5730 years) ($5.421 x 10^{11}$s)
4. The half life of pure C_s - 137 is 35 years.
5. A sample of radioactive element decays into a stable nuclide with decay constant λ. If N_s is the number of stable atoms at a time t and N_t is the number of radioactive atoms remaining at a time t;

(i) Show that $\lambda = \frac{1}{t}\log_e(\frac{N_s + N_t}{N_t})$

(ii) Sketch on the same graph, time variations of N_s and N_t.

6. Living wood has an activity of 16.0countsmin^{-1}g^{-1} which is due to the disintegration of carbon – 14 atoms in the wood. The half life of carbon –14 is 5566years.
   a) Calculate the number of carbon atoms in 1g of living wood.(4.3.10.22)
   b) If the sample of mass 0.5g from the ancient ship wreckage is found to have an activity of 6.5countsmin^{-1}, find the age of the ship.(1667.4)
7. A sample of radioactive material contains 10^{14} atoms. The half life of the material is 2.5days. calculate:
   a. The fraction remaining after 4 days.(0.33)
   b. The activity of the sample after 4 days.(2.08.10.8)
8. A radioactive source has a half life of 25s and an initial activity of 7.2x10^{12}Bq. Calculate its activity after 60s have elapsed.(1.36.10.12)
9. A radioactive isotope X is present as 1% of the atoms of a monoatomic element Y, which has a relative atomic mass of 80. Assuming the half life of X is 5hours. Calculate the rate of decay of 10g of Y after 10 hours.(9.6.10._6)
10. An isotope of krypton ($^{87}_{36}Kr$)has a half life of 78 minutes. Calculate the activity of 10μg of$^{87}_{36}Kr$. (Avogadro's number = 6.02x10^{23}mol^{-1})(1.02.10.13)
11. A radioactive source contains 1μg of protonium of mass number 239. The source is estimated to emit 2300 alpha particles per second in all directions. Calculate the half life of Protonium.(7.53.10.11)
12. A radioactive source produces alpha particles each of energy 60MeV. If 20% of the alpha particles enter an ionization chamber a current of 0.2μA flows. Find the activity of alpha source, if the energy needed to make an ion pair in the chamber is 32MeV.
13. A radioactive isotope of strontium of mass 5μg has a half life of 28years. "Find the mass of the 0 left after 14 years. (Assume decay law).(3.5)
14. A radioisotope 60_{CO} decays by emission of a beta particle and gamma rays. Its half life is 5.3years. Find the activity of a source containing 0.20g of60_{CO}.(8.29.10.12)
15. Uranium 238U has a half life of 4.5x10^9years. If the Earth solidified about 4.0x10^9years, what fraction of 238U then found on the Earth remains un decayed today? (0.5)
16. (i) What is meant by half of a radioactive material?

   (ii) Given that radioactive law, $N_t = N_0 e^{-\lambda t}$, obtain the relation between λ and half life, $\frac{T_1}{2}$.

   (iii) What are radio – isotopes?

(iv) The radio – isotopes $^{90}_{38}Sr$ decays by emission of beta particles, the half life of radio – isotopes is 28.8 years. Determine the activity of 1g of the Isotope.(5.1*1012)

17. (a) what is meant by the following terms *radioactivity, half-life, and decay constant*

(b) The activity of a sample of dead wood is 10 counts per minute while activity of a living plant is 19counts/min. if the half life of C-14 is 5500yrs, find the age of the sample of wood (assume that $A = A_0 e^{-\lambda t}$)

18. *A sample of radioactive material contains 10^8 atoms. The half-life of the material is 2 days. Calculate*

(i) The fraction remaining after 5 days

(ii) The activity of the sample after 5 days.

19. Find the activity of 1g sample of radium $^{226}_{88}Ra$ whose half life is 1620 yrs

20. The nuclide $^{124}_{55}Cs$ decays with a half life is 30.8seconds. If we have $7.5\mu g$ initially,

a. How many nuclei are present initially? (**ANS** 3.64×10^{16} *nuclei*)

b. How many nuclei are present 2.0 min later? $(2.45\times10^{15}\,nuclei)$

c. What is the activity 2.0 min later? $(5.51\times10^{13}\,Bq)$

d. After how long will the activity fall to less than one per second? $(1526s)$

Measurement of half life of a radioactive material

a. **Long lived radioactive material**

A Geiger Muller tube is used.

Procedure

i. A sample of a radioactive material is weighed and it's mass, m, determined. If the relative atomic mass of the material is M, then the number of atoms is given by $N = \frac{m}{M} x 6.02 x 10^{23}$.

ii. The rate of emission, $\frac{dN}{dt}$, all round the material is determined by placing it at a distance r, from the end face, area A, of the G.M tube connected to a scaler.

iii. If the measured count rate through the area A,isC,then $\frac{\left(\frac{dN}{dt}\right)}{C} = \frac{4\pi r^2}{A}$, where $\frac{dN}{dt} = -\lambda N$, the decay constant λ can be calculated as N and $\frac{dN}{dt}$ are known. The half life is then obtained from $T_{\frac{1}{2}} = \frac{0.693}{\lambda}$.

Example:

1. A G.M tube placed 10cm from 2g of $^{222}_{86}Rn$ gives a count rate of 75counts per second. If the widow of the G.M tube has an area of 5cm^2, calculate:
   i. Number of atoms disintegrating per second.
   ii. Half life of Rn in years.

   Solution

b. **Short lived radioactive material**

This can be done using an ionization chamber or a Geiger muller tube

(i) An ionization chamber is used.

Procedure

i. A bottle with a radioactive material in gaseous form is connected to the ionization chamber.
ii. The radioactive gas is passed into the chamber by squeezing the bottle.
iii. When the atoms decay, they produce radiations which ionize the gas in the chamber thus electrons move to the anode and positive ions to the cathode. This results into flow of current which is registered by the d.c amplifier.
iv. Starting with appreciable deflection on the d.c amplifier, the flowing current is noted at equal time intervals.
v. A graph of In I against t is then plotted from the results and a straight line graph is obtained from which the slope is calculated. The slope is the decay constant.
vi. The gradient of the graph is the decay constant. The half life is calculated from $T_{\frac{1}{2}} = \frac{0.693}{\lambda}$.

Theory:

$I = I_0 e^{-\lambda t}$. Introducing In on both sides, In I = In $I_0 - \lambda t$

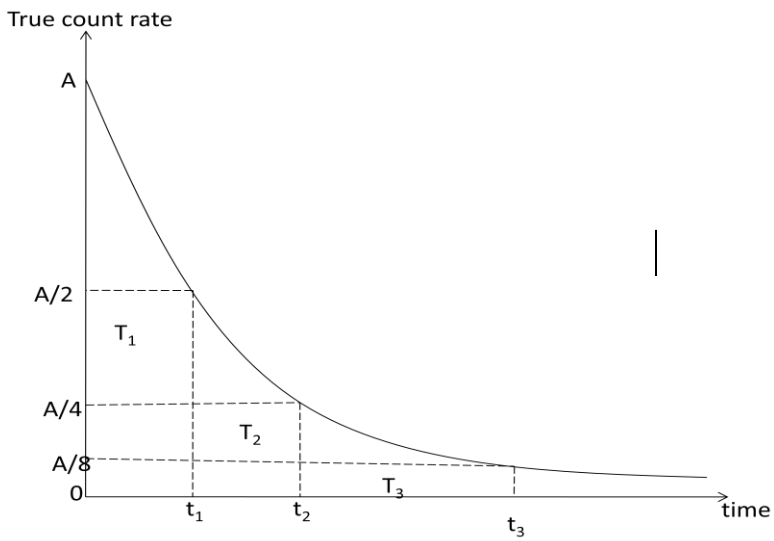
(ii) Geiger Muller tube

The GM tube is switched on and the background count rate A_o is noted and recorded.

The source of ionizing radiation is placed near the window of the GMT window and the count rate recorded at equal time intervals.

For each count rate recorded, the background count rate is subtracted off to get the true count rate.

A graph of true count rate against time is plotted.



The time T_1 taken for the activity to reduce to A/2, T_2 for the activity to reduce to A/4 and T_3 for the activity to reduce to A/8 is found.

The half life $= \frac{1}{2}(T_1 + T_2 + T_3)$

Example

1. The following table shows some measurements of the decay rate of a radioactive sample. Find the disintegration constant.

Time (min)	In R (Bq)
36	5.08
100	3.29
164	1.52
218	1.00

Uses of radioactive material

a) Industrial use

i. Checking the thickness of materials during manufacturing process for example paper and plastic. This is done by having alpha particle sources below the material, a radioactive detector above the count rate will be found as proportional to the thickness of the material and therefore the desired thickness can be obtained corresponding to a given count rate.

ii. Used as level indicator for example filling bottles in bear and beverage factories.

iii. The ionizing effect can be used to reduce static electricity which develops on machinery due to leak of charge.

b) Biological uses

i. Used in radiotherapy; gamma rays can be used in treatment of cancer cells (tumor).

ii. Sterilization of medical equipments.

iii. They are used in food preservation.

iv. They are used by archeologists to determine the ages of rocks.

v. They are used in thickening of oils.

vi. They are used in vulcanation of rubber.

Carbon dating

Is the process of determining the age of dead plants or animals using the isotope of carbon14.

Process of carbon dating

A radioactive form of carbon ${}^{14}_{6}C$ is present in all plants. The plants may be eaten by animals including human beings. Its amount is constant in all living things. It is formed as a result of neutrons bombarding the nuclei of nitrogen in the atmospheres.

$${}^{14}_{7}N + {}^{1}_{0}\cap \rightarrow {}^{14}_{6}C + {}^{1}_{1}H$$

Living matter take in carbon in form of CO_2 from the atmosphere. When a plant dies it stops taking in CO_2 and its ${}^{14}C$ decays to ${}^{14}N$by β particle emission.

By measuring the activity of ${}^{14}C$ in the remains, the time when the plant died can be estimated. The process of finding the age of fossils is called carbon dating.

Question:

Carbon – 14 decays by emission of beta particles.

a. Explain what happens in the nucleus of carbon – 14 during decay process.
b. Write an equation for the decay,
c. Outline the use of carbon – 14 in carbon dating.

ASSIGNMENT

1. $^{23}_{11}Na$ is a *radioactive isotope* of sodium which has a *half-life* period of 15hours and disintegrates with the emission of alpha particles and gamma-rays. It emits alpha particles that have energies of 4.2 MeV.)
Explain the meaning of the terms that are italicized in the statement above.
2. Give concisely the important facts about mass, charge and velocity associated with alpha, beta and gamma radiations respectively. State the effect, if any, of the emission of each of these radiations on (a) the mass number and (b) the atomic number, of the element concerned.
3. 'Gamma rays obey the inverse square law.' What does this mean? What conditions must be satisfied for the statement to be valid?
Describe an experiment designed to verify the statement for gamma rays in air.
The window area of a gamma ray detector is 50cm^2 . The window is placed horizontally and lies 80 cm vertically below a small source of gamma rays, 60 photons per minute from the source are incident on it. Estimate the rate of emission of photons from the source.
4. Describe the nature of alpha, beta and gamma radiations.
What is meant by the statement that the stable isotope of gold has an atomic number of 79 and a mass number of 197? A sample of pure gold is irradiated with neutrons to produce a small proportion of the radioactive isotope of gold of mass number 198. What experiments would you perform to examine the radiation emitted by the sample to establish whether it was alpha, beta or gamma radiation?
If chemical analysis of the sample subsequently showed that it contained a trace of mercury (atomic number 80) what would you conclude from this about the nature of the radiation from the radioactive gold? What would you expect the mass number of the isotope of mercury present in the gold to be?
5. A Geiger-Muller tube is placed close to a source of beta particles of constant activity. Sketch a graph showing how the count-rate, measured using a suitable scaler or rate meter, varies with the potential difference applied to the G.M. tube.
Discuss how the form of the graph determines the choice of operating conditions for the tube.
Describe how you would investigate the absorption of beta particles of aluminium using a G.M. tube. Sketch a graph showing the results you would expect to obtain. How would the form of the graph change if
(a) the same source was used with lead substituted for aluminium,
(b) a different source emitting beta particles of higher energy was used, aluminium being the absorbing material?
6. Compare and contrast the properties of α-particles, protons and neutrons.

Discuss briefly the part played by the two last-named particles in atomic structure.

Compare the velocities attained by a proton and an alpha-particle each of which has been accelerated from rest through the same potential difference.

7. In nuclear fusion, deuterium nuclei might fuse together to form a single helium nucleus. If the atomic masses of deuterium and helium are 2.010 and 4.004 a.m.u respectively, and 1 a.m.u. = 931 MeV, calculate the energy released in MeV.

8. What is gamma-radiation? Explain one way in which it originates.

10. Using the information on atomic masses given below, show that a nucleus of uranium 238 can disintegrate with the emission of an alpha particle according to the reaction

$^{238}U \rightarrow ^{234}Th + ^{4}He$.

Calculate (a) the total energy released in the disintegration, (b) the kinetic energy of the alpha particle, the nucleus being at rest before disintegration, .Mass of 238U = 238.12492 a.m.u. Mass of 234Th = 234.1 1650 a.m.u. Mass of4He = 4.00387 a.m.u. 1 a.m.u. (atomic mass unit) is equivalent to 930 MeV.

9. Compare and contrast the properties of the proton, neutron and electron. Explain the role played by each of these particles in the structure of the atom. How is your account of the arrangement of the electrons in the atom supported by experimental evidence?

10. The decay of a certain radioactive source is found to be exponential. Explain this statement.

The disintegration of a radioactive atom is considered to be a matter of chance. How do you reconcile this idea with the observed regular law of decay?

To determine the half fife of the radioactive gas thoron an experimenter uses an ionization chamber into which he introduces air loaded with thoron. The RADIOACTIVITY, THE NUCLEUS 1067ionization current is 9.0 x 10^{-12} A initially and falls to half of this value in 60 seconds, which is taken to be the half life of thoron. Unknown to the experimenter, the ionization chamber is contaminated, from previous use, with a radioactive substance whose half life is 11 hours, which is responsible for 4 x 10^{-12} A of the ionization current at the outset of the experiment. Calculate a more accurate value for the half life of thoron.

What experimental procedure would have revealed this source of error?

11. What is meant by the half-life period (half-life) of a radioactive material?

Describe how the nature of α-particles has been established experimentally. The half-life period of the body polonium-210 is about 140 days. During this period the average number of alpha emissions per day from a mass of polonium initially equal to 1 microgram is about 1.2×10^{12}. Assuming that one emission takes place per atom and that the approximate density of polonium is 10 g cm^{-3}, estimate the number of atoms in 1cm^3 of polonium.